SOLUTIONS AND ANSWERS
TO
SHOP MATHEMATICS

BY

JOHN M. CHRISTMAN

FORMERLY WITH FORD MOTOR COMPANY AND WITH
WILLYS-OVERLAND COMPANY AS MATHEMATICAL
SUPERVISOR AND SUPERVISOR OF TRAINING

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SOLUTIONS AND ANSWERS

EXERCISE 1

1. 5, 7. **2.** 2, 29. **3.** 2, 2, 2, 3, 5. **4.** 3, 41.
5. 3, 3, 41. **6.** 2, 2, 3, 5, 7. **7.** 37. **8.** 29, 29.
9. 27, 59. **10.** 39, 41. **11.** 83, 99. **12.** 81, 97.
13. 99, 99.

EXERCISE 2

1. 3. **2.** $2-125-8+200-24=2+200-125-8-24=202-157=45$. **3.** $2+8\times3-7\times9+100\div2=2+24-63+50=13$.
4. 525. **5.** 25. **6.** 2. **7.** 8. **8.** 4.

EXERCISE 3

1. $\frac{5}{7}$. **2.** $\frac{1}{2}$. **3.** $2\frac{3}{22}$. **4.** $15\frac{26}{27}$. **5.** $\frac{56}{19}$.
6. $\frac{327}{4}$. **7.** $\frac{18}{22}\frac{15}{35}=\frac{605}{745}=\frac{121}{149}$. **8.** $\frac{15}{20}=\frac{45}{60}$. 45. *Ans.*
9. $\frac{36}{63}=\frac{12}{21}=\frac{48}{84}$. 84. *Ans.*
10. $\frac{21}{25}\frac{60}{20}=\frac{216}{252}=\frac{54}{63}=\frac{6}{7}=\frac{42}{49}$. 49. *Ans.*

EXERCISE 4

1. $4\,\Big|\,\dfrac{12,\ 5,\ 4}{3,\ 5,\ 1}$. L.C.M. $=4\times3\times5\times1=60$.

2. $\frac{3}{8}+\frac{5}{6}+\frac{7}{9}+\frac{1}{4}=\frac{27}{72}+\frac{60}{72}+\frac{56}{72}+\frac{18}{72}=\frac{161}{72}=2\frac{17}{72}$.

3. $\frac{7}{8}+2\frac{1}{2}+\frac{6}{5}+\frac{3}{2}+17=\frac{35}{40}+2\frac{20}{40}+\frac{48}{40}+\frac{60}{40}+17=19\frac{163}{40}=23\frac{3}{40}$.

4. $\frac{1}{10}+\frac{1}{9}+\frac{1}{8}+\frac{1}{7}=\frac{252}{2520}+\frac{280}{2520}+\frac{315}{2520}+\frac{360}{2520}=\frac{1207}{2520}$.

5. $\frac{5}{12}-\frac{3}{10}=\frac{25}{60}-\frac{18}{60}=\frac{7}{60}$.

6. $\frac{1}{8}-\frac{1}{9}=\frac{9}{72}-\frac{8}{72}=\frac{1}{72}$.

7. $\frac{3}{4}+\frac{3}{5}=\frac{15}{20}+\frac{12}{20}=\frac{27}{20}$.
$\frac{11}{35}+\frac{1}{12}=\frac{132}{420}+\frac{35}{420}=\frac{167}{420}$.
$\frac{27}{20}-\frac{167}{420}=\frac{567}{420}-\frac{167}{420}=\frac{400}{420}=\frac{20}{21}$.

8.

9073 | 16247	1
7174 | 9073	1
1899 | 7174	3
1477 | 5697	1
422 | 1477	3
422 | 1266	2
0 | 211	

$\dfrac{9073\div211}{16247\div211}=\dfrac{43}{77}$.

9. $7\frac{17}{126}$.

10. $\frac{3}{8}+\frac{49}{64}+\frac{7}{16}+\frac{11}{32}=\frac{24}{64}+\frac{49}{64}+\frac{28}{64}+\frac{22}{64}=\frac{123}{64}=1\frac{59}{64}$.

11. $\frac{11}{7}+\frac{2}{7}=\frac{13}{7}=1\frac{6}{7}$.

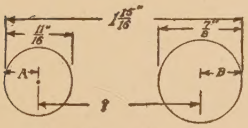

FIG. 4S

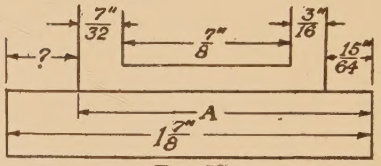

FIG. 5S

12. $A=1\frac{11}{16}''\div 2=1\frac{11}{32}''$.
$B=\frac{7}{8}''\div 2=\frac{7}{16}''$.
$?=1\frac{15}{16}''-\frac{7}{16}''-\frac{11}{32}''=1\frac{5}{32}''$.

13. $A=\frac{7}{32}''+\frac{7}{8}''+\frac{3}{16}''+\frac{15}{64}''=\frac{14}{64}''+\frac{56}{64}''+\frac{12}{64}''+\frac{15}{64}''=\frac{97}{64}''=1\frac{33}{64}''$.
$?=1\frac{7}{8}''-1\frac{33}{64}''=1\frac{56}{64}''-1\frac{33}{64}''=\frac{23}{64}''$.

EXERCISE 5

1. $\frac{\overset{3}{\cancel{6}}\times\overset{2}{\cancel{8}}\times\cancel{9}\times\cancel{10}}{\cancel{8}\times\cancel{3}\times\cancel{5}}=36$.

2. $\frac{\overset{3}{\cancel{48}}\times\overset{3}{\cancel{72}}\times\overset{4}{\cancel{8}}}{\underset{2}{\cancel{32}}\times\cancel{24}\times\underset{3}{\cancel{27}}}=\frac{4}{3}=1\frac{1}{3}$.

3. $6\frac{66}{85}$.

4. $\frac{3}{1280}$.

5. 7.

6. 36.

7. 1.

8. $\frac{7\times 8-3+8}{8+3\times 9-5}=\frac{56-3+8}{8+27-5}=\frac{61}{30}=2\frac{1}{30}$.

9. $\frac{7\times 11-102+7\times 4}{8+7\times 3-17}=\frac{77-102+28}{8+21-17}=\frac{3}{12}=\frac{1}{4}$.

10. $6\frac{2}{3}$.

11. $5\frac{19}{48}$.

12. $\frac{7\times 4+6-9\div 3}{8\div 2+4}=\frac{28+6-3}{4+4}=\frac{31}{8}=3\frac{7}{8}$.

13. $1773\frac{1}{9}$.

14. $88\frac{7}{9}$.

15. $7\times\frac{43}{64}''+2\times\frac{1}{2}''=\frac{301}{64}''+1''=5\frac{45}{64}''$.

16. $\frac{9}{10}$ of a foot $=\frac{9}{10}\times 12=10\frac{4}{5}''$. If $\frac{5}{6}$ is turned, $\frac{1}{6}$ is unfinished. $\frac{1}{6}$ of $10\frac{4}{5}''=\frac{1}{6}\times\frac{54}{5}''=\frac{9}{5}''=1\frac{4}{5}''$. $1\frac{4}{5}$ inches. *Ans.*

EXERCISE 6

1. $\frac{\frac{7}{8}\times 8}{\frac{3}{4}\times 8}=\frac{7}{6}=1\frac{1}{6}$.

2. $\frac{\frac{5}{12}\times 12}{9\times 12}=\frac{5}{108}$.

3. $\frac{15\frac{3}{4}}{16\frac{2}{3}}=\frac{\frac{63}{4}\times 12}{\frac{50}{3}\times 12}=\frac{189}{200}$.

4. $18\frac{2}{3}$.

EXERCISE 7

1. $\frac{1}{72}$.

2. 6.

3. $1\frac{15}{17}$.

4. $\frac{2}{57}$.

5. $\frac{\frac{7}{12}}{\frac{2}{5}}=1\frac{11}{24}$.

6. $54\div\frac{6}{7}=54\times\frac{7}{6}=63$.

7. $\frac{7}{9}\div\frac{14}{6}=\frac{7}{9}\times\frac{6}{14}=\frac{1}{3}$.

8. $34\frac{2}{5}\div 46\frac{2}{7}=\frac{172}{5}\div\frac{324}{7}=\frac{172}{5}\times\frac{7}{324}=\frac{301}{405}$.

9. $\frac{5\frac{7}{16}''-(6\times\frac{1}{8}'')}{7}=\frac{5\frac{7}{16}''-\frac{3}{4}''}{7}=\frac{4\frac{11}{16}''}{7}=4\frac{11}{16}''\times\frac{1}{7}=\frac{75}{16}''\times\frac{1}{7}=\frac{75}{112}''$.

EXERCISE 8

1. ~~$24\times\frac{35}{24}=35$~~. $\frac{35}{24}$. *Ans.*

2. $\frac{4}{25}$.

3. ~~$\frac{5}{6}\times\frac{\frac{9}{281}}{\frac{5}{6}}=\frac{9}{281}$~~. $\frac{\frac{9}{281}}{\frac{5}{6}}$. *Ans.*

4. $\frac{11\frac{1}{2}}{8\frac{2}{3}}$.

5. $\frac{\frac{17}{4}}{\frac{2}{3}}$.

6. $\frac{\frac{4}{5}}{7}$.

7. $\frac{6}{\frac{5}{7}}$

8. $\frac{2\times\frac{7}{6}}{6\times\frac{7}{6}}=\frac{2\frac{1}{3}}{7}$. $2\frac{1}{3}$. *Ans.*

9. $\frac{\frac{3}{4}}{\frac{5}{6}}=\frac{9}{10}$; $\frac{\frac{11}{10}}{\frac{5}{12}}=\frac{66}{25}$; then $\frac{\frac{3}{4}}{\frac{5}{6}}\times\frac{\frac{11}{10}}{\frac{5}{12}}=\frac{9}{10}\times\frac{66}{25}=\frac{297}{125}$.

10. $\frac{78\frac{1}{3}}{\frac{9}{12}}=\frac{940}{9}$; $\frac{2\frac{8}{9}}{3}=\frac{26}{27}$; then $\frac{78\frac{1}{3}}{\frac{9}{12}}\div\frac{2\frac{8}{9}}{3}=\frac{940}{9}\div\frac{26}{27}=\frac{940}{9}\times\frac{27}{26}=\frac{14\,10}{13}=108\frac{6}{13}$.

11. $\frac{47\frac{2}{3}\times\frac{6\frac{2}{3}}{\frac{9}{14}}}{\frac{9}{14}\times\frac{6\frac{2}{3}}{\frac{9}{14}}}=\frac{?}{6\frac{2}{3}}$; $47\frac{2}{3}\times\frac{6\frac{2}{3}}{\frac{9}{14}}=\frac{143}{3}\times\frac{20}{3}\times\frac{14}{9}=\frac{40040}{81}=494\frac{26}{81}$; $494\frac{26}{81}$. *Ans.*

EXERCISE 9

1. 338.9158.

2. 4000.0004.

3. $4.675+.4+.045+.87+18.11=24.10$.

4. 12.080.

5. 11.78509. **6.** .0782. **7.** 34.202. **8.** .999″.

9. $1.872'' + .7165'' - 1.031'' = 1.5575''$.

10. $.9025'' - .502'' + .801'' - .502'' = .6995''$.

11. $\frac{1.125''}{2} - .183'' + \frac{.904''}{2} = .8315''$.

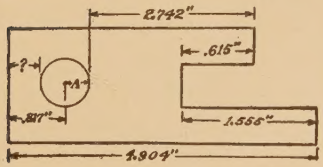

FIG. 13S

12. $A = 4.904'' - .817'' - 1.555'' + .615'' - 2.742'' = .405''$.

$? = .817'' - .405'' = .412''$.

13. 2.5845″. **14.** $\frac{5.503'' - (4 \times .241'')}{5} = .9078''$. **15.** 1.439″.

16. 20.187. **17.** .0057. **18.** .0894. **19.** 59.31.

20. 16.677. **21.** $\frac{6.6}{6.7} + \frac{87}{100} - .934 = .985 + .87 - .934 = .921$.

22. $\frac{9.06 \times \frac{13.2}{9.06}}{.054 \times \frac{13.2}{9.06}} = \frac{13.2}{.0786}$.

23. .624″ *Ans.*

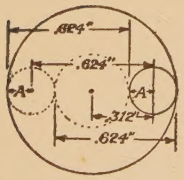

FIG. 17S

24. $\frac{1}{2} \times 1.125'' - \frac{4}{9} \times 1.125'' = .0625''$.

25. .5384. **26.** $.3875 = \frac{3875 \div 125}{10000 \div 125} = \frac{31}{80}$.

27. $\frac{18.6}{47.43} = \frac{1860 \div 93}{4743 \div 93} = \frac{20}{51}$. **28.** .5456.

29. $\frac{4.444}{8.674}$ or .51233.

30. $\frac{17\frac{2}{3}}{23\frac{5}{7}} = \frac{3\,7\,1}{4\,9\,8} = .74497$. **31.** 2.2043. **32.** 1.6667.

33. $\frac{14 \times .8}{17.4 \div 6} = \frac{?}{18}$

$\frac{11.2}{2.9} = \frac{?}{18}$

$\frac{11.2 \times \frac{18}{2.9}}{2.9 \times \frac{18}{2.9}} = \frac{69.51}{18}$.

69.51 *Ans.*

34. 2.5548″.

EXERCISE 10

1.
54326 = 2
64832 = 5
434608 10 = 1
1738432
3476864
3522063232 = 1

2.
62541 = 0
54515 = 2
312705 0
938115
2814345
3409422615 = 0

3.
68952 = 3
48896 = 8
551616 24 = 6
3309696
6619392
3371476992 = 6

4.
712653 = 6
560749 = 4
4988571 24 = 6
34919997
39908568
399619457097 = 6

5.
70425 = 0
998 = 8
70425000 0
140850
70284150 = 0

6.
40365 = 0
499 = 4
20182500 0
40365
20142135 = 0

7. $7241 \div 3125 = \frac{7241}{3125} = \frac{.07241}{.03125} = \frac{.07241}{\frac{1}{32}} = .07241 \times 32$

.07241 = 5
32 = 5
14482 25 = 7
21723
2.31712 = 7

8. 4 | 965200
241300

EXERCISE 11

1. 1.36 = 1%, then 17 = $12\frac{1}{2}$%. **2.** $258\frac{3}{4}$ lb. of copper.

3. .4%. **4.** 44. **5.** 2883.9 lb. **6.** $\frac{1}{4}$ of $1\frac{1}{2}\% = \frac{3}{8}\%$;

$\frac{3}{8}$% of 2200 lb. $= \frac{66.00 \text{ lb.}}{8} = 8.25$ lb.

7. \$.0095 = 1%; \$2.25 − \$0.95 = \$1.30 = Increase in price;
\$1.30 ÷ .0095 = 136.8%.

8. 2500 parts in 24 hours equals $104\frac{1}{6}$ parts in 1 hour.
1500 " " 10 " " 150 " " 1 "

This makes an increase of $45\frac{5}{6}$ parts in 1 hour.

If $104\frac{1}{6}$ equals 100%, $45\frac{5}{6}$ equals 44%. 44% increase. *Ans.*

9. $12\frac{1}{2}\%$ of 96 is 12; 12 is $33\frac{1}{3}\%$ of 36. **10.** .0096″.

11. A = .750375″. B = .749625″.

EXERCISE 12

1. $42-(9+6)+2=42-15+2=44-15=29$. **2.** 8.

3. 46. **4.** 107. **5.** $(16-2)(17+9)=14\times26=364$. **6.** 4.

7. $5\left(\frac{15-6}{2(4+3)}\right)-\frac{34-2}{16}=5\left(\frac{9}{14}\right)-\frac{32}{16}=\frac{45}{14}-2=1\frac{3}{14}$.

8. $2+\{14-(6+2)\}\{(6+2)(18+2)-90\}-6\div2=2+\{14-8\}\{(8)(20)-90\}-3=2+6(160-90)-3=2+6\times70-3=2+420-3=419$.

EXERCISE 13

1. $\frac{3(2+B)+ABC-3}{CD-(BC-A)+3B}=\frac{3(2+2)+6\times2\times8-3}{8\times8-(2\times8-6)+3\times2}=$

$\frac{12+96-3}{64-(16-6)+6}=\frac{105}{60}=\frac{7}{4}=1\frac{3}{4}$.

2. 10825″. **3.** 282.744 cubic inches. **4.** .054125″.

5. 1.1686″. **6.** 4. **7.** $1\frac{7}{16}$″. **8.** 76.9692 cubic inches.

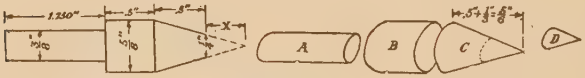

FIG. 29S

9. $X=\frac{\frac{1}{4}\times.5}{\frac{5}{8}-\frac{1}{4}}=\frac{1}{3}$.

A = $.7854(\frac{3}{8})^2(1.750)$ = .1932 cubic inch.

B = $.7854(\frac{5}{8})^2(.5)$ = .1534 cubic inch.

C = $1.0472(\frac{5}{16})^2(\frac{5}{6})$ = .0852 cubic inch.

D = $1.0472(\frac{1}{8})^2(\frac{1}{3})$ = .0054 cubic inch.

Volume = A + B + C − D = .1932 + .1534 + .0852 − .0054 = .4264 cubic inch.

10. 2.298″. **11.** 2.348″. **12.** .100.

EXERCISE 14

1. $26\times\frac{4}{7}=14\frac{6}{7}$ teeth. **2.** $7\times\frac{70}{81}=6\frac{4}{81}$ revolutions.

3. $\frac{4}{9}\times\frac{81}{70}=\frac{18}{35}$ revolution. **4.** A = 37 teeth; B = 55 teeth.

5. $\dfrac{A=\frac{4}{7}\text{ revolution}}{B=\frac{11}{18}\text{ revolution}}=\dfrac{A=\frac{11}{18}\text{ teeth.}}{B=\frac{4}{7}\text{ teeth.}}=\dfrac{11}{18}\times\dfrac{7}{4}=\dfrac{77}{72}$ 77 teeth in A.
" " B.

6. $\frac{11}{13}=\frac{66}{78}$, A = 78 teeth.

7.

1537	2597	1
1060	1537	1
477	1060	2
424	954	4
53	106	2
	106	
	0	

$$\frac{1537\div 53}{2597\div 53}=\frac{29}{49}.$$

29 teeth and 49 teeth. *Ans.*

8. $\frac{56}{88}=\frac{7}{11}=\frac{35}{55}$, X = 35 teeth. **9.** $3\times 12\times .200''=7.2''$.

EXERCISE 15

1. After factoring (A = 1739 = 37×47; B = 1541 = 23×67) and placing in proper compound order the following sets of compound gears are formed.

C = 37; D = 23; E = 47; F = 67.
C = 37; D = 67; E = 47; F = 23.
C = 47; D = 23; E = 37; F = 67.
C = 47; D = 67; E = 37; F = 23.

2. C and E are the factors of A; therefore, if A = 4753 and C = 49, E must equal $\frac{47}{49}\frac{53}{}$ or 97 teeth.

D and F are the factors of B; therefore, if B = 1333 and F = 43, D must equal $\frac{13}{43}\frac{33}{}$ or 31 teeth.

3. After obtaining suitable factors

$$\left(\frac{X}{Y}=\frac{360}{47}=\frac{4\times 90}{1\times 47}=\frac{(25\times 4)\times 90}{(25\times 1)\times 47}=\frac{100\times 90}{25\times 47}\right)$$

and placing in proper compound order, the following sets of compound gears are obtained.

A = 100; B = 47; C = 90; D = 25.
A = 100; B = 25; C = 90; D = 47.
A = 90; B = 47; C = 100; D = 25.
A = 90; B = 25; C = 100; D = 47.

4. After factoring ($1092=28\times39$; $1224=24\times51$) and placing in proper compound order, the following sets of compound gears are formed.

$A=28;\ B=24;\ C=39;\ D=51.$
$A=28;\ B=51;\ C=39;\ D=24.$
$A=39;\ B=24;\ C=28;\ D=51.$
$A=39;\ B=51;\ C=28;\ D=24.$

5. $\frac{36\times42}{88\times C}=\frac{63}{187}=\frac{9\times7}{11\times17}=\frac{(4\times9)\times7}{(4\times11)\times17}=\frac{(4\times9)\times(2\times7)}{(2\times4\times11)\times17}=$
$\frac{(4\times9)\times(3\times2\times7)}{(2\times4\times11)\times(3\times17)}=\frac{36\times42}{88\times51}$. C = 51 teeth. *Ans.*

6. After obtaining suitable factors
$\left(\frac{X}{Y}=\frac{71}{403}=\frac{1\times71}{13\times31}=\frac{(3\times1)\times71}{13\times(3\times31)}=\frac{(7\times3\times1)\times71}{(7\times13)\times(3\times31)}=\frac{21\times71}{91\times93}\right)$
and placing in proper compound order, the following sets of compound gears are obtained.

$A=21;\ B=91;\ C=71;\ D=93.$
$A=21;\ B=93;\ C=71;\ D=91.$
$A=71;\ B=91;\ C=21;\ D=93.$
$A=71;\ B=93;\ C=21;\ D=91.$

7. $\frac{A}{B}=\frac{35\times64}{25\times40}=\frac{(35\div5)\times(64\div8)}{(25\div5)\times(40\div8)}=\frac{7\times8}{5\times5}=\frac{56}{25}$, A = 56 teeth; B = 25 teeth. *Ans.*

8. $\frac{6.455}{1}\times\frac{22}{17}\times\frac{25}{26}=8.0322$ revolutions.

9. If A′ makes 203 revolutions while D′ makes 121 revolutions, A′ must have 121 teeth and D′ 203 teeth. After obtaining suitable factors
$\left(\frac{A'}{D'}=\frac{121}{203}=\frac{11\times11}{7\times29}=\frac{(3\times11)\times(2\times11)}{(3\times7)\times(2\times29)}=\frac{33\times22}{21\times58}\right)$

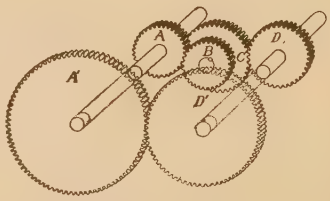

FIG. 60S

and placing in proper compound order, the following sets of compound gears are obtained.

A=33; B=21; C=22; D=58.
A=33; B=58; C=22; D=21.
A=22; B=21; C=33; D=58.
A=22; B=58; C=33; D=21.

10. 8 inches is equal to 40 teeth in rack. When rack moves 40 teeth, X will make $40 \div 20$ or 2 revolutions. When A makes 4 revolutions, X must make 2 revolutions. The number of revolutions inverted equals the number of gear teeth. Thus, A=2 teeth and X=4 teeth. A must be multiplied by 15 to equal 30, therefore X must be multiplied by the same number. $X = 4 \times 15$ or 60 teeth.

EXERCISE 16

1. $4\frac{2}{3} \times \frac{1}{7}'' = \frac{2}{3}''$.

2. $8.5 \times \frac{1}{7}'' = 1.21428''$.

3. $1.703 \div \frac{1}{7}'' = 11.921''$.

4. While A makes $5\frac{1}{5}$ revolutions, the screw will make $5\frac{1}{5} \times \frac{36}{45}$ or $\frac{104}{25}$ revolution. One revolution of screw moves slide $\frac{1}{5}''$; then $\frac{104}{25}$ revolution will move it $\frac{104}{25} \times \frac{1}{5}'' = \frac{104}{125}'' = .832''$.

5. Screw must make $4.362 \div \frac{1}{5}$ or 21.81 revolutions to move slide $4.362''$. While screw makes 21.81 revolutions, A will make $21.81 \times \frac{45}{36}$ or 27.2625 revolutions.

6. While A makes 1 revolution, B will make $\frac{76}{81}$ or .93827 revolution.

7. While A makes 2 revolutions, B will make $2 \times \frac{57}{39} \times \frac{72}{30}$ or 7.0153 revolutions.

8. While B makes 2 revolutions, A will make $2 \times \frac{30}{72} \times \frac{39}{57}$ or .57017 revolution.

9. B must make $.355 \div \frac{1}{5}$ or 1.775 revolutions to move slide $.355''$. While A makes 1 revolution, B makes 1.775 revolutions, therefore A must have 1775 teeth and B 1000 teeth or A=71 teeth and B=40 teeth. PROOF: $\frac{71}{40} \times \frac{1}{5}'' = .355''$.

10. While crank makes $1\frac{5}{8}$ revolutions, the worm wheel will make $1\frac{5}{8} \times \frac{20}{41} \times \frac{2}{51}$ or .03108 revolution.

11. While A makes 1 revolution, B will make $1 \times \frac{40}{1} \times \frac{53}{67}$ or 31.641 revolutions.

EXERCISE 17

1. 1.0993. **2.** 28. **3.** $\frac{13}{24}$. **4.** $\frac{\$11.43}{1}\times\frac{4}{9}=\$5.08$.

5. $23''\times\frac{3.52}{4.56}=17.75''$. **6.** $\frac{17}{64}$. **7.** $100\times\frac{54}{29}=186.2$ revolutions.

8. $72\times\frac{12}{18}=48$ teeth. **9.** $.236''\times\frac{\frac{7}{9}}{\frac{4}{5}}=.236''\times\frac{7}{9}\times\frac{5}{4}=.2294''$.

EXERCISE 18

1. $\$12.50\times\frac{7}{4}\times\frac{6}{5}=\$26.25$. **2.** $7\times\frac{4}{12}\times\frac{40}{26}=3.59$ hours.

3. $45\times\frac{1}{2}\frac{12}{50}\times\frac{7}{3}=47.04$ pounds. **4.** $\frac{20539}{1}\times\frac{10}{8}\times\frac{4}{3}\times\frac{3}{2}=$ 51347.5 pounds. **5.** $1000\times\frac{18}{6}\times\frac{16}{8}=6000$ revolutions.

6. $100\times\frac{31}{12}\times\frac{45}{29}=400.8$ revolutions. **7.** $5\times\frac{80}{71}\times\frac{1}{6}''=.93896''$.

8. $50\times\frac{24}{11}\frac{0}{5}=104.34$ revolutions.

9. $6\times\frac{37}{20}\times\frac{25}{31}=8.9516$ revolutions.

EXERCISE 19

1. While A makes 1 revolution, the table is elevated $.250''\times\frac{2}{5}$ or $.100''$. If one revolution elevates it $.100''$, to elevate it $.001''$ will require $.001\div.100$ or .01 of a revolution. Therefore, the collar must have 100 divisions so that each division will cause A to revolve .01 of a revolution. 100 divisions. *Ans.*

2. While A makes 1 revolution, the slide moves $1\times.01\times.1''$ or $.001''$. If 1 revolution of A makes the slide move .001 of an inch, to move it .0001 of an inch would require $.0001\div.001$ or .1 of a revolution. Therefore the collar A must have 10 divisions so that each division will move it .1 of a revolution. 10 divisions. *Ans.*

3. One revolution of the cross-slide screw moves the slide $.250''$. If collar A has 125 divisions, each division will move it $.002''$. When the slide moves $.002''$, the diameter of the shaft is reduced $.004''$. If one division on the collar reduces the shaft diameter $.004''$, to reduce it $.018''$ it must be revolved $.018\div.004$ or 4.5 divisions. $4\frac{1}{2}$ divisions. *Ans.*

4. While A makes 1 revolution, the slide moves $.0375''$. If 1 revolution of A moves the slide $.0375''$, to move it $.0005''$ would require $.0005\div.0375$ or $\frac{1}{75}$ of a revolution. Therefore, A must have 75 divisions so that each division will move it $\frac{1}{75}$ of a revolution. 75 divisions. *Ans.*

5. While A moves 3 notches, the screw makes $\frac{3}{50}\times\frac{52}{26}$ or $\frac{3}{25}$ of a revolution. Since 1 revolution of the screw moves the table $\frac{2}{5}''$, $\frac{3}{25}$ of a revolution will move it $\frac{2}{5}''\times\frac{3}{25}$ or .048″. *Ans.*

6. While the rack moves 1 inch, C makes 1 revolution, and D makes 10 revolutions, because each revolution of D moves the rack .100″. When D makes 10 revolutions, A makes $10\times\frac{15}{60}$ or $2\frac{1}{2}$ revolutions; therefore, while the rack moves 1 inch, C makes 1 revolution, and A makes $2\frac{1}{2}$ revolutions.

If we know the number of revolutions the simple gears (A and C) must make, the numbers of teeth in them are obtained by inverting the number of the revolutions. Thus, if A makes $2\frac{1}{2}$ revolutions while C makes 1 revolution, A must have 1 tooth and B $2\frac{1}{2}$ teeth. B must be multiplied by 24 to obtain 60; therefore, A must also be multiplied by 24 to keep the ratio the same. Then A equals 24×1 or 24. 24 teeth. *Ans.*

EXERCISE 20

1. 213. **2.** 1010. **3.** .750. **4.** 47.138. **5.** .06.

6. 3.00001. **7.** 28.536. **8.** $\sqrt{\frac{32.1}{16.07}}=\sqrt{1.99751}=1.4133.$

EXERCISE 21

1. 175.5 minutes or 2 hours, 55 minutes, 30 seconds.

2. $\frac{45}{\frac{1}{4}}\times\frac{42}{11}$ or 687.2 R.P.M. **3.** 120 revolutions.

4. 21.48 R.P.M.

5. 5342 feet per minute. **6.** 15.71 feet per minute.

7. $\frac{6000}{6}\times\frac{42}{11}=3818$ R.P.M.; $B=10''\times\frac{910}{3818}=2.383''$.

8. When spindle makes 1 revolution, the spindle feeds

$$1\times\tfrac{45}{15}\times\tfrac{3}{8}\times\tfrac{20}{40}\times\tfrac{1}{130}\times12\times.3142''=\frac{8.4834''}{520}=.0163''.$$

To have a cutting speed of 60 feet per minute drill must make $\frac{60}{2}\times\frac{42}{11}$ or 114.5 R.P.M. To drill a hole 5″ deep by using a feed of .0163″, drill must make 306.7 revolutions. For the drill to make 306.7 revolutions at 114.5 R.P.M. it would take 2.67 minutes.

9. To have a cutting speed of 60 feet per minute, a 3.750″ shaft must make $\frac{60}{3.750}\times\frac{42}{11}$ or 61+ revolutions. The cutting is not to

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9. To have a cutting speed of 60 feet per minute, a 3.750″ shaft must make $\frac{60}{3.750}\times\frac{42}{11}$ or 61+ revolutions. The cutting is not to

exceed 60 feet per minute; therefore, the lathe would run at 44 R.P.M. on first cut. By using a .030″ feed the lathe would have to make $\frac{10.25}{.030}$ or 341.6 revolutions to finish the first cut on the shaft. At 44 R.P.M. it would take $\frac{341.6}{44}$ or 7.76 minutes to reduce shaft to 3.5″.

The lathe would have to make 65+ R.P.M. to have a cutting speed of 60 feet per minute on a 3.5″ shaft; therefore, on the second cut the lathe would also run 44 R.P.M., and the cutting time would be 7.76 minutes to reduce the shaft from 3.5″ to 3.25″.

The lathe would have to make 70+ R.P.M. to have a cutting speed of 60 feet per minute on a 3.25″ shaft; therefore, on the next cut the lathe would run at 68 R.P.M. By using a .030″ feed the lathe would have to make 208.3 revolutions to reduce both ends of the shaft to 3″. At 68 R.P.M. it would take 3.06 minutes to do this.

The total time on shaft would then be 7.76+7.76+3.06 or 18.58 minutes.

10. Crank must make $\frac{45}{2\times 2}\times\frac{42}{11}$ or 42.95 R.P.M. for tool to have a maximum speed of 45 feet per minute. While crank makes 42.95 R.P.M., A will make $42.95\times\frac{60}{12}$ or 214.75 R.P.M.

EXERCISE 22

1. $?=180°-123°\ 14'=56°\ 46'$.

2. $?=180°-(40°+58°+31°)=51°$.

3. $?=32°-25°+7°=14°$.

4. $90°-37°=53°$.

5. $A=90°-50°=40°$.
$?=70\frac{1}{2}°-40°=30\frac{1}{2}°$.

Fig. 115S

6. $?=64°+59°=123°$.

7. $?=\frac{180°-50°}{2}=65°$.

Fig. 118S

8. $A=60°-29°=31°$.
$B=60°$.
$?=180°-(60°+31°)=89°$.

EXERCISE 23

1. $5.674'' \times 3.1416 = 17.825''$.

FIG. 124S

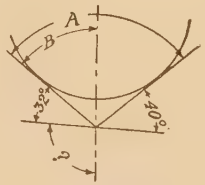


FIG. 125S

2. $A = \frac{38}{2}° = 19°$.
$B = \frac{56}{2}° = 28°$.
$? = 180° - (19° + 28°) = 133°$.

3. $A = 180° - (40° + 32°) = 108°$.
$B = \frac{108}{2}° = 54°$.
$? = 180° - (54° + 32°) = 94°$.

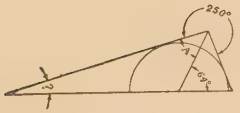


FIG. 126S

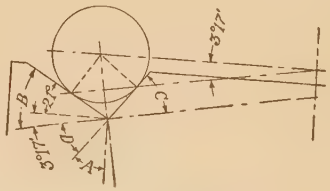


FIG. 127S

4. $A = \dfrac{360° - 250°}{2} = 55°$.
$? = 64° - 55° = 9°$.

5. $B = 24°\ 17'$. $C = 24°\ 17'$.
$D = 24°\ 17'$, because it is an opposite angle to C.
$A = 90° - (24°\ 17' + 3°\ 17') = 62°\ 26'$.

EXERCISE 24

1. $1.375'' \times 1.4142 = 1.9445''$.

2. $3.5'' \div 1.4142 = 2.4749''$.

3. $1.125'' \times 1.1547 = 1.29903''$.

4. $1.700'' \div 1.1547 = 1.4722''$.

5. $180° - 2(128° - 90°) = 104°$.

6. The sum of the interior angles $= (6-2)180° = 720°$. $A = 720° - (44° + 300° + 42° + 142° + 72°) = 720° - 600° = 120°$.

7. $X = 3.1416'' \div 10 = .31416''$. Circumference of small gear $= 1.2'' \times 3.1416$. Then $(1.2'' \times 3.1416) \div .31416'' = 12$, the number of teeth in the small gear.

EXERCISE 25

1. $A = \frac{5}{8}'' \times \frac{5}{8}'' \div 1\frac{5}{16}'' = .2976''$.
$? = 1\frac{5}{16}'' + .2976'' = 1.6101''$.

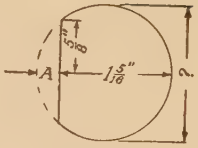


FIG. 148S

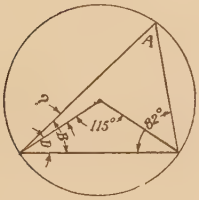


FIG. 149S

2. $D = \frac{180° - 115°}{2} = 32\frac{1}{2}°$.
$A = 1\frac{1}{2}5 = 57\frac{1}{2}°$.
$B = 180° - (57\frac{1}{2}° + 82°) = 40\frac{1}{2}°$.
$C = 40\frac{1}{2}° - 32\frac{1}{2}° = 8°$.

3. $? = 16'' + 12'' - 8'' = 20''$. **4.** $? = 2\,(180° - 80° - 26°) = 148°$.

EXERCISE 26

1. $10'' \times 5.4'' = 54$ square inches. **2.** 45.36 square inches.

3. 10.455 square inches. **4.** 34.16 square inches.

5. $\frac{3.1'' + 9.2''}{2} \times 4.1'' = 25.215$ square inches.

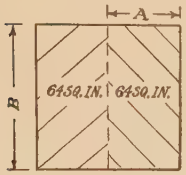


FIG. 169S

6. 103.869 square inches.

7. $\pi R^2 = 12.76$ square inches.

$$R^2 = \frac{12.76}{\pi} = 4.06162.$$

$R = \sqrt{4.06162} = 2.015''$.
Diameter $= 2 \times 2.015'' = 4.030''$.

8. $B = \sqrt{128} = 11.313''$.

9. $X = \frac{160}{5} = 32$ square inches.
$Y = 2 \times 32 = 64$ square inches.
$A = \sqrt{64} = 8''$.

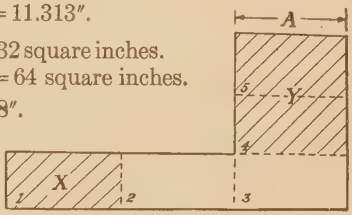


FIG. 170S

10. Area of a 14″ square = 196 square inches. Area of a 14″ circle = 153.9384 square inches. The difference = 42.0616 square inches, and the area of A = 42.0616 ÷ 4 = 10.515 square inches.

11. $4\times6''\times6''\times3.1416 = 452.3904$ square inches.

EXERCISE 27

1. $7''\times5''\times7.3'' = 255.5$ cubic inches. **2.** 126.9632 cubic inches. **3.** $3.1''\times3.1''\times3.1416\times7.1'' = 214.354$ cubic inches.

4. 103.90842 cubic inches. **5.** $\frac{1}{2}(4''\times7.1'')\ (\frac{1}{3})\ (5.2'') = 24.613$ cubic inches. **6.** $4''\times4''\times3.1416\times\frac{1}{3}\times6.2'' = 103.882$ cubic inches.

7. $\dfrac{2''+4''}{2}\times6''\times\frac{1}{3}\times4.2'' = 25.2$ cubic inches.

8. $\frac{4}{3}\times7''^3\times3.1416 = 1436.7584$ cubic inches.

EXERCISE 28

1. $C = \sqrt{10.11^2+6.72^2} = 12.139$. **2.** $C = \sqrt{11^2+14^2} = 17.804$.

3. $A = \sqrt{7.2^2-6.4^2} = 3.2984$. **4.** $B = \sqrt{8.04^2-.312^2} = 8.0339$.

5. $A = \sqrt{10.7^2-5.661^2} = 9.0798$.

6. $? = 2\sqrt{2^2-(\frac{7}{8})^2} = 2\times\sqrt{3.234375} = 3.5968''$.

7. $? = (1\frac{3}{8}+1\frac{1}{4}) - \sqrt{(1\frac{3}{8})^2-1} = 2.625''-.9437'' = 1.6813''$.

8. $? = \sqrt{1^2-.5^2} = .86603''$.

9. $? = .375+.250+\sqrt{(4.5-.500-.250)^2+(3.750-.500-.375)^2}$
$= .625+\sqrt{22.3281} = .625''+4.7252'' = 5.3502''$.

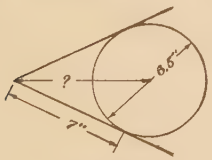


FIG. 193S

10. $? = \sqrt{3.25^2+7^2}$
$? = \sqrt{10.5625+49}$
$? = \sqrt{59.5625}$
$? = 7.7177''$.

11. $2.600+1.62-\sqrt{2.6^2+1.62^2} =$
$4.22-\sqrt{6.76+2.6244} =$
$4.22-\sqrt{9.3844} =$
$4.22''-3.0634'' = 1.1566''$. Diameter.

12. $B=\sqrt{.8^2-.250^2}=$
$\sqrt{.64-.0625}=$
$\sqrt{.5775}=$
$.7599''.$
$?=.8''+.7599''-.250''=$
$1.3099''.$

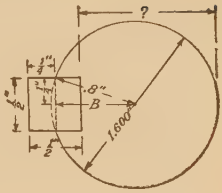


FIG. 195S

13. $A=4''-.9''=3.1''.$
$B=4''+\frac{7}{8}''=4\frac{7}{8}''.$
$C=\sqrt{A^2+B^2}.$
$C=\sqrt{3.1^2+4.875^2}=$
$\sqrt{9.61+23.7656}=$
$\sqrt{33.3756}=5.7771''.$
$?=5.7771''+.9''+.875''=$
$7.5521''.$

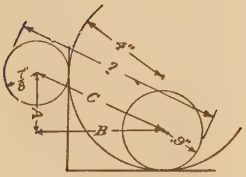


FIG. 196S

14. $?=\sqrt{6^2+2.7^2}=6.5795''.$

15. $A=\sqrt{7^2+7^2}=9.8994''.$
$B=9.8994''-7''=2.8994''.$
$C=2B=5.7988''.$
$D=1.4142''\times 2.8994''=4.1003''.$
$?=2\times 4.1003''-5.7988''=$
$2.4018''.$

FIG. 198S

EXERCISE 29

1. $?=\dfrac{8.4^2-5.2^2}{2\times 10.5}=2.0723.$

2. $?=\dfrac{12.6^2-9.3^2}{2\times 6}=6.0225.$

3. $Y=\dfrac{7.21^2-5.9^2}{2\times 8.2}=1.0472''.$
$X=4.1''-1.0472''=$
$3.0528''.$
$?=\sqrt{5.9^2-3.0528^2}=$
$5.0488''.$

FIG. 204S

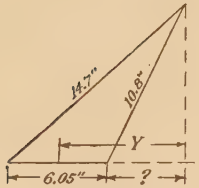


FIG. 205S

4. $Y=\dfrac{14.7^2-10.8^2}{2\times 6.05}=8.219''.$

$?=8.219''-3.025''=5.194''.$

EXERCISE 30

1. 44°. **2.** $\dfrac{180^\circ}{8}=22\frac{1}{2}^\circ$. **3.** 17° 56′ 42″. **4.** 79° 22′ 45″.

5. $26^\circ\ 37'\ 23''=26^\circ\ 37\frac{23}{60}'=\left(26\dfrac{37\frac{23}{60}}{60}\right)^\circ=26.623^\circ$

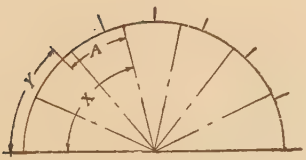


FIG. 216S

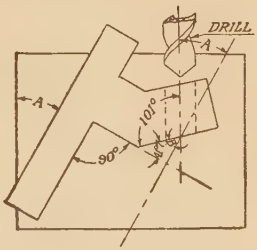


FIG. 217S

6. $X=\frac{3}{7}\times 180^\circ=77\frac{1}{7}^\circ$.

$Y=\frac{2}{8}\times 180^\circ=45^\circ$.

$A=X-Y=77\frac{1}{7}^\circ-45^\circ=32\frac{1}{7}^\circ$.

$A=32^\circ\ 8'\ 43''$.

7. $B=90^\circ-41^\circ\ 7'=48^\circ\ 53'$.

$A=180^\circ-(101^\circ+48^\circ\ 53')=30^\circ\ 7'$.

8. $168^\circ=\frac{168}{360}$ or $\frac{7}{15}$ of a revolution; $172^\circ=\frac{172}{360}$ or $\frac{43}{90}$ of a revolution. By inverting the revolutions and calling them gear teeth, $A=\frac{43}{90}$ of a tooth and $B=\frac{7}{15}$ of a tooth. By multiplying both by 90, A = 43 teeth and B = 42 teeth.

9. $123^\circ\times\frac{48}{37}=159.567^\circ=159^\circ\ 34'\ 1''$.

10. Circumference $=3.1416\times 6.35''=19.949''$. $\dfrac{3.271}{19.949}$ = part of the whole circle that θ equals. $\theta=360^\circ\times\dfrac{3.271}{19.949}=59.028^\circ$ or $59^\circ\ 1'\ 41''$.

11. $\text{Circumference} = 1.865'' \times \frac{360^\circ}{62^\circ 14'} = 1.865'' \times \frac{360}{62\frac{7}{30}} = 10.788''.$

$\text{Diameter} = \frac{10.788}{3.1416} = 3.4339''.$

EXERCISE 31

1. $A = 1.317'' - .892'' = .425''.$

FIG. 229S

2. $X = 2.575'' - 1.725'' = .850''$. $Y = 7 \times .024'' = .168''$. $B = .850'' + .168'' = 1.018''$.

3. $C = 2.545'' - .892'' = 1.653''$.

4. $D = (1.300'' - .150'') - (14 \times .024'') = .814''$.

5 $A = 1.014''$.

6. $B = (1.350'' - .475'') - (.024'' \times 7) = .707''$.

7. $A = .284''$.

8. $B = .289''$.

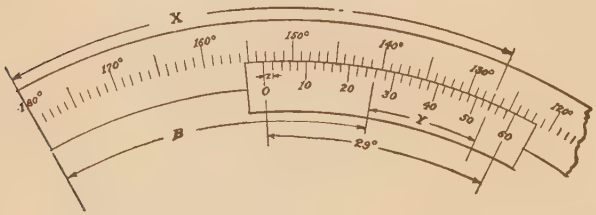


FIG. 232S (a)

9. $A = 26^\circ\ 54'$.

10. $X = 53^\circ$. $Z = \frac{29}{30}^\circ$.
$Y = 14 \times \frac{29}{30}^\circ = 14 \times 58' = 13^\circ\ 32'$.
$B = 53^\circ - 13^\circ\ 32' = 39^\circ\ 28'$.

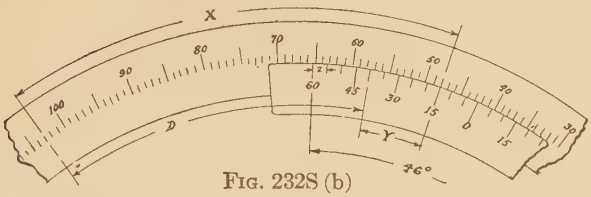


FIG. 232S (b)

11. C = 42° 15′.

12. X = 55°.
Z = $\frac{4\frac{6}{24}}{4}$° = $1\frac{1}{12}$° = 1° 55′.
Y = 5 × 1° 55′ = 9° 35′.
D = 55° − 9° 35′ = 45° 25′.

EXERCISE 32

1. $? = .601'' \times \frac{1.764}{.913} = 1.1611''$.

2. $? = 3.1416 \times \frac{.687}{1} = 2.1583$.

3. $A = \sqrt{.75^2 + .5^2} = .90138''$.

$? = .90138'' \times \frac{2}{.750} = 2.4036''$.

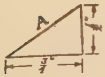


FIG. 242S

4. $? = .875 \times \frac{1.1}{2.31} = .416666$.

5. $? = .4 \times \frac{1.33}{.871} = .61079''$.

6. $? = 1'' \times \frac{1}{.8571} = 1.1667''$.

7. $? = 8.1'' \times \frac{4}{9} = 3.6''$.

8. $A = \sqrt{2.41^2 + 1.32^2} = 2.7478$.

$? = 1.32'' \times \frac{1.71}{2.7478} = .82145''$.

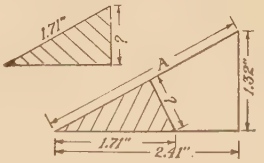


FIG. 247S

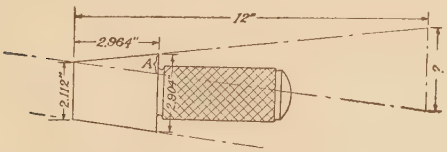


FIG. 248S

9. $A = 2.904'' - 2.112'' = .792''$.

$? = .792'' \times \frac{12}{2.964} = 3.2064''$.

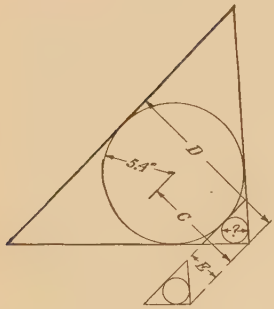


FIG. 249S

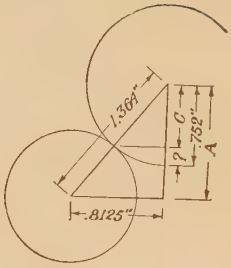


FIG. 250S

10. $C = 1.4142 \times 5.4'' = 7.6366''$.

$D = 7.6368'' + 5.4'' = 13.0366''$.

$E = 7.6368'' - 5.4'' = 2.2366''$.

$? = 10.8'' \times \frac{2.2366}{13.0366} = 1.8529''$.

11. $A = \sqrt{1.364^2 - .8125^2} = 1.0956''$.

$C = 1.0956'' \times \frac{.752}{1.364} = .60402''$.

$? = .752'' - .60402'' = .14798''$.

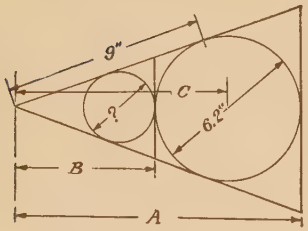


FIG. 251S

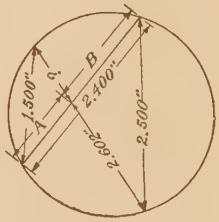


FIG. 252S

12. $C = \sqrt{9^2 + 3.1^2} = 9.5189''$.

$A = 9.5189'' + 3.1'' = 12.6189''$.

$B = 9.5189'' - 3.1'' = 6.4189''$.

$? = 6.2'' \times \frac{6.4189}{12.6189} = 3.1537''$.

13. $A = 2.602'' \times \frac{1.5}{2.5} = 1.5612''$.

$B = 2.400'' - 1.5612'' = .8388''$.

$? = .8388'' \times \frac{1.5}{2.5} = .50328''$.

EXERCISE 33

FIG. 258S

1. $A = 1.18'' - .7'' = .48''$.

$$B = .48'' \times \frac{12}{1\frac{3}{4}} = \frac{23.04''}{7} = 3.2914'' = \text{taper per foot.}$$

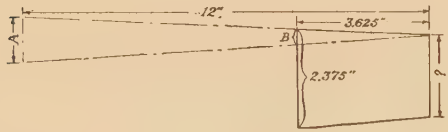


FIG. 259S

2. $A = .4375''$ or taper per foot.

$$B = .4375'' \times \frac{3.625}{12} = \frac{1.5859''}{12} = .1321''.$$

$$? = 2.375'' - .1321'' = 2.2429''.$$

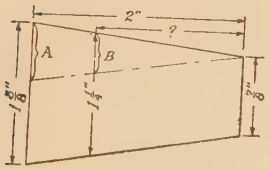


FIG. 260S

3. $A = 1\frac{3}{8}'' - \frac{7}{8}'' = \frac{4}{8}'' = \frac{1}{2}''$.

$B = 1\frac{1}{4}'' - \frac{7}{8}'' = \frac{3}{8}''$.

$$? = 2 \times \frac{\frac{3}{8}}{\frac{1}{2}} = 1.5''.$$

4. $A = 3\frac{3}{8}'' - 1\frac{3}{4}'' = 1\frac{5}{8}''$.

$B = 1\frac{5}{8}'' \times \frac{1\frac{7}{8}}{4\frac{1}{4}} = .7169''$.

$? = 1.750'' + .7169'' = 2.4669''$.

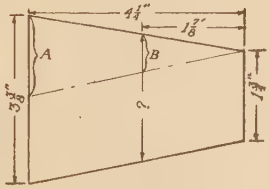


FIG. 261S

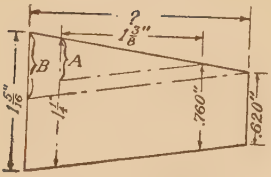


FIG. 262S

5. $A = 1.250'' - .760'' = .490''$.
$B = 1.3125'' - .620'' = .6925''$.
$? = 1\frac{3}{8}'' \times \frac{.6925}{.490} = 1.9432''$.

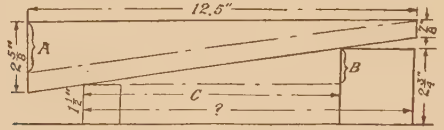


FIG. 263S

6. $A = 2\frac{5}{8}'' - \frac{7}{8}'' = 1\frac{3}{4}''$. $B = 2\frac{3}{4}'' - 1\frac{1}{2}'' = 1\frac{1}{4}''$.
$C = 12.5'' \times \frac{1\frac{1}{4}}{1\frac{3}{4}} = 8.9285''$.
$? = 8.9285'' + 2.750'' = 11.6785''$.

7. $A = 1'' - \frac{3}{8}'' = \frac{5}{8}''$.
$B = 5\frac{7}{8}'' \times \frac{.009}{\frac{5}{8}} = .0846''$.

If one revolution of screw moves gib $\frac{1}{12}$ of an inch, to move it .0846″ would require $.0846 \div \frac{1}{12}$ or 1.0152 revolutions.

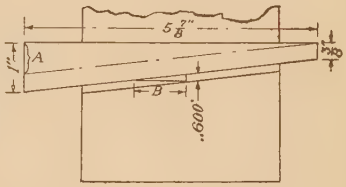
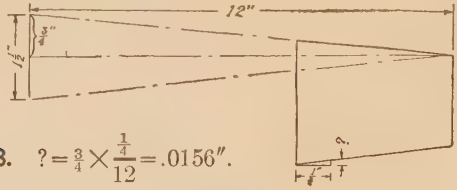


FIG. 264S



8. $? = \frac{3}{4} \times \frac{\frac{1}{4}}{12} = .0156''$.

FIG. 265S

EXERCISE 34

1. $A = 10 \times .25882 = 2.5882$.
2. $B = 10 \times .96592 = 9.6592$.
3. $A = 8 \times 1.0075 = 8.06$.
4. $B = 8 \times .12278 = .98224$.
5. $A = 2.175 \times 1.7434 = 3.7919$.
6. $B = 2.175 \times 1.4281 = 3.1061$.
7. $A = 4.6947 \div 10 = .46947$.
8. $B = 8.8295 \div 10 = .88295$.
9. $A = 2.3877 \div 2.025 = 1.179$.
10. $B = 1.2654 \div 2.025 = .62488$.
11. A, E, I. **12.** B, D, H. **13.** C, F, G.

EXERCISE 35

1. Sine. **2.** Cosine. **3.** Tangent. **4.** Secant.
5. Cotangent. **6.** Cosecant. **7.** A = secant 23° = 1.0864.
8. B = tangent 23° = .42447. **9.** A = sine 29° = .48481.
10. B = cosine 29° = .87462. **11.** A = cosecant 20° 3′ = 2.9168.
12. B = cotangent 20° 3′ = 2.7400. **13.** A = secant 46° = 1.4395.
14. B = tangent 46° = 1.0355. **15.** A = sine 59° = .85717.
16. B = cosine 59° = .51504. **17.** A = cosecant 16° = 3.6279.
18. B = cotangent 16° = 3.4874.

EXERCISE 36

1. A = 4.3 (secant 16°) = 4.3 × 1.0403 = 4.47329.
2. B = 4.3 (tangent 16°) = 4.3 (.28674) = 1.23298.
3. C = 4.3 (sine 16°) = 4.3 (.27564) = 1.18525.
4. A = 1.760 (cotangent 31°) = 1.760 (1.6643) = 2.929168.
5. B = 1.760 (cosecant 31°) = 1.760 (1.9416) = 3.4172.
6. A = 1.01 (cosine 42° 11′) = 1.01 (.74100) = 7.4841.

EXERCISE 37

1. 2.3428 = cosecant 25° 16′. **2.** 2.7450 = tangent 69° 59′.
3. 3.1379 = secant 71° 25′. **4.** 2.4525 = cotangent 22° 11′.
5. 1.1699 = secant 31° 16′. **6.** .76754 = sine 50° 8′.
7. .52646 = cosine 58° 14′. **8.** 3.0595 = tangent 71° 54′.

EXERCISE 38

1. Cotangent $A = \frac{2.170}{\frac{3}{4}} = 2.89333$, A = 19° 3′ 59″; or tangent A = $\frac{\frac{3}{4}}{2.170} = .345622$, A = 19° 3′ 59″. **2.** Secant $A = \frac{12.021}{10.5} = 1.14485$, A = 29° 8′ 8″; or cosine $A = \frac{10.5}{12.021} = .87347$, A = 29° 8′ 8″.

3. A = .375″ (cotangent $17\frac{1}{2}°$) = .375″ (3.1716) = 1.18935″.

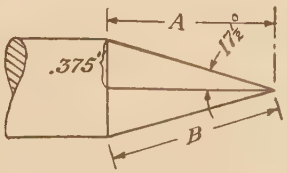


Fig. 314S

4. B = .375″ (cosecant $17\frac{1}{2}°$) = .375″ (3.3255) = 1.24707″.

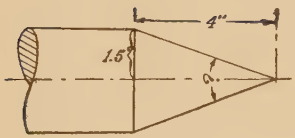


FIG. 315S

5. Tangent of $\frac{1}{2}$ the angle $= \frac{1.5}{4} =$ .375; $\frac{1}{2}$ angle $= 20° 33' 21''$; or cotangent of $\frac{1}{2}$ the angle $= \frac{4}{1.5} = 2.6667$; $\frac{1}{2}$ angle $=$ $20° 33' 21''$.
$? = 2(20° 33' 21'') = \mathbf{41° 6' 42''}$

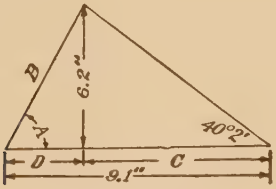


FIG. 316S

6. $C = 6.2''(\text{cotangent } 40° 2') = 6.2''(1.1903) = 7.37986''$.
$D = 9.1'' - 7.37986'' = 1.72014''$.
Cotangent $A = \frac{1.72014}{6.2} = .27744$. $A = 74° 29' 38''$.

7. $B = 6.2''(\text{cosecant } 74° 29' 38'') = 6.2''(1.03778) = 6.4343''$; or
$B = \sqrt{D^2 + 6.2^2} = \sqrt{1.72014^2 + 6.2^2} = \sqrt{2.9589 + 38.44} = \sqrt{41.3989} = 6.4343''$.

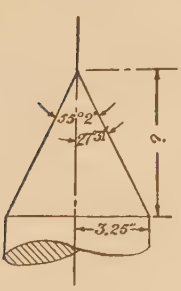


FIG. 317S

8. $? = 3.25''$ (cotangent $27° 31'$) $= 3.25''(1.9196) =$ $6.2387''$.

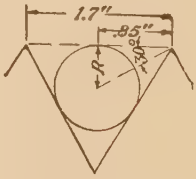


FIG. 318S

9. $R = .850''(\text{tangent } 30°) =$ $.850''(.57735) =$ $.49074''$.
Diameter $= 2(.49074'') =$ $.98148''$.

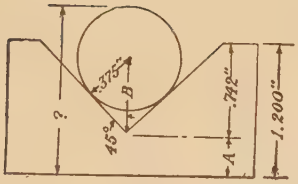


FIG. 319S

10. A = 1.200″ − .742″ = .458″.
B = .375″(cosecant 45°) = .375″(1.4142) = .53033″.
? = .458″ + .53033″ + .375″ = 1.3633″.

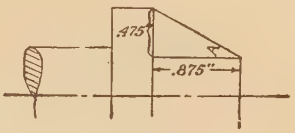


FIG. 320S

11. Cotangent $A = \frac{.875}{.475} =$ 1.8421.
A = 28° 29′ 45″.
? = 2(28° 29′ 45″) = 56° 59′ 30″.

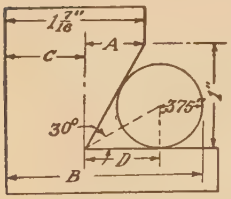


FIG. 321S

12. A = 1″(tangent 30°) = .57735″.

13. C = 1.4375″ − .57735″ = .86015″
D = .375″ (cotangent 30°) = .375″ × 1.7321 = .64954″.
B = .86015″ + .64954″ + .375″ = 1.8847″.

15. C = 1.125″.
A = 1.125″ − .250″ = .875″.
D = 1.125″(sine 60°) = 1.125″(.86603) = .97428″.

16. B = .97428″ + .1875″ − .125″ = 1.03678″.

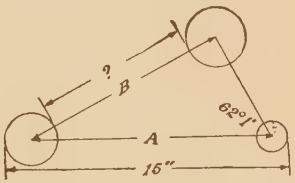


FIG. 322S

14. A = 15″ − .750″ − .4375″ = 13.8125″.
B = A(sine 62° 1′) = 13.8125″(.88308) = 12.1975″.
? = 12.1975″ − .750″ − .9375″ = 10.510″.

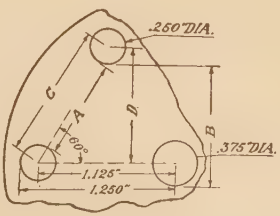


FIG. 323S

EXERCISE 39

1. Tangent A = 1.75 ÷ 3.45 = .50725. A = 26° 53′ 47″.
Cotangent A = 3.45 ÷ 1.75 = 1.9714. A = 26° 53′ 47″.

2. X = 3.45(secant 26° 53′ 47″) = 3.45 × 1.1213 = 3.8685.
X = 1.75(cosecant 26° 53′ 47″) = 1.75 × 2.2106 = 3.8685
$X = \sqrt{3.45^2 + 1.75^2} = 3.8685$.

3. Sine A = 2.67 ÷ 4.006 = .66650. A = 41° 47′ 52″.
Cosecant A = 4.006 ÷ 2.67 = 1.50037. A = 41° 47′ 52″.

4. X = 4.006(cosine 41° 47′ 52″) = 4.006 × .74551 = 2.9864.
X = 2.67(cotangent 41° 47′ 52″) = 2.67 × 1.1185 = 2.9864.
$X = \sqrt{4.006^2 - 2.67^2} = 2.9864$.

5. Cosine A = (5.62″ − 1″) ÷ (6.01″ − 1″) = .92215.
A = 22° 45′ 27″.

6. X = 1″ + (5.62″ − 1″)(tangent 22° 45′ 27″) = 1″ + 4.62″ × .41949
= 2.938″, or $X = 1'' + \sqrt{(6.01-1)^2 - (5.62-1)^2} = 2.938''$.

7. Tangent A = (1.1″ + 2″) ÷ (8.52″ − 2.50″) = .51495.
A = 27° 14′ 47″.

8. X = (8.52″ − 2.50″)(secant 27° 14′ 47″) − 1.5″ =
6.02″ × 1.1248 − 1.5″ = 5.2713″, or
$X = \sqrt{(8.52-2.50)^2 + (1.1+2)^2} - 1.5'' = 5.2713''$.

EXERCISE 40

FIG. 332S

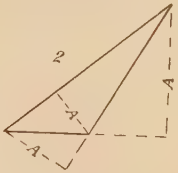


FIG. 332AS

FIG. 332BS

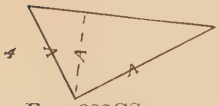


FIG. 332CS

FIG. 332DS

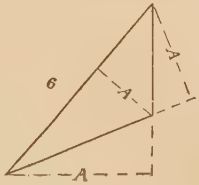


FIG. 332ES

FIG. 332FS

EXERCISE 41

1. $X=7+3=10$. **2.** $X=9-3.5=5.5$. **3.** $X=180°-84°=96°$.

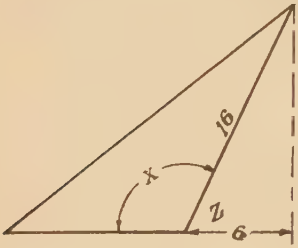


FIG. 339S

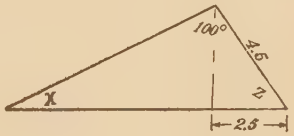


FIG. 340S

4. Cosine $Z=6\div16=.375$.
$Z=67°\ 58'\ 32''$.
$X=180°-67°\ 58'\ 32''=112°\ 1'\ 28''$.

5. Cosine $Z=2.5\div4.5=.55555$.
$Z=56°\ 15'\ 4''$.
$X=180°-(100°+56°\ 15'4'')=23°\ 44'\ 56''$.

6. $X=\dfrac{4^2-3^2}{2\times6}=\dfrac{7}{12}=.58333$. **7.** $X=8+\dfrac{14^2-12^2}{2\times16}=8+\dfrac{52}{32}=9.625$.

8. $X=\dfrac{5.5^2-3^2}{2\times3.5}=\dfrac{21.25}{7}=3.0357$.

9. $X=1.25+\dfrac{5.25^2-3.5^2}{2\times2.5}=1.25+\dfrac{15.3125}{5}=4.3125$.

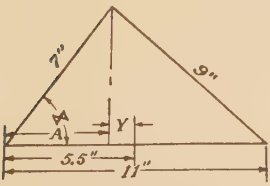


FIG. 345S

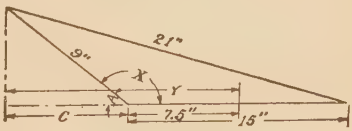


FIG. 346S

10. $Y=\dfrac{9^2-7^2}{2\times11}=\dfrac{81-49}{22}=\dfrac{32}{22}=1.4545''$.
$A=5.5''-1.4545''=4.0455''$.
Cosine $X=\dfrac{4.0455}{7}=.57793$.
$X=54°\ 41'$.

11. $Y=\dfrac{21^2-9^2}{2\times15}=\dfrac{441-81}{30}=12''$.
$C=12''-7.5''=4.5''$.
Cosine $A=\dfrac{4.5}{9}=.5$. $A=60°$.
$X=180°-60°=120°$.

EXERCISE 42

1. X = 5.5(sine 30°) = 5.5 × .50000 = 2.75.

2. X = 21 − 14.5 = 6.5.

3. X = 8.5(cosine 27°) = 8.5 × .89101 = 7.5735.

4. X = 7.2(cosine 42°) = 7.2 × .74314 = 5.3506.

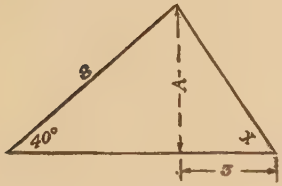


FIG. 356S

5. A = 8(sine 40°) = 8 × .64279 = 5.14232.
Tangent X = 5.14232 ÷ 3 = 1.714141.
X = 59° 44′ 27″.

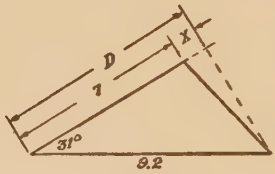


FIG. 357S

6. D = 9.2(cosine 31°) = 9.2 × .85717 = 7.8859.
X = 7.8859 − 7 = .8859.

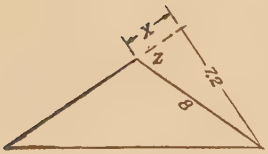


FIG. 358S

7. Sine Z = 7.2 ÷ 8 = .90000.
Z = 64° 9′ 30″.
X = 8(cosine 64° 9′ 30″) = 8 × .43588 = 3.487, or
X = $\sqrt{8^2 - 7.2^2}$ = 3.487.

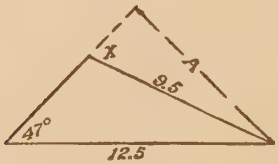


FIG. 359S

8. A = 12.5(sine 47°) = 12.5 × .73135 = 9.1419.
Sine X = 9.1419 ÷ 9.5 = .96230.
X = 74° 13′.

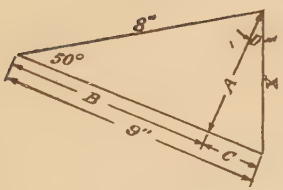


FIG. 360S

9. A = 8″(sine 50°) = 8″(.76604) = 6.12832″.
B = 8″(cosine 50°) = 8″(.64279) = 5.14232″.
C = 9″ − 5.14232″ = 3.85768″.
Cotangent $D = \frac{6.12832}{3.85768} = 1.58860.$
D = 32° 11′ 24″.
X = 6.12832″(secant 32° 11′ 24″) = 6.12832″(1.1816) = 7.2412″.

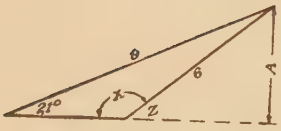


FIG. 361S

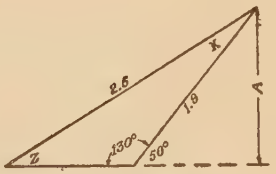


FIG. 362S

10. A = 9(sine 21°) = 9 × .35837 = 3.22533.
Sine Z = 3.2253 ÷ 6 = .53755.
Z = 32° 31′ 3″.
X = 180° − 32° 31′ 3″ = 147° 28′ 57″.

11. A = 1.9(sine 50°) = 1.9 × .76604 = 1.45547.
Sine Z = 1.45547 ÷ 2.5 = .58219.
Z = 35° 36′ 17″.
X = 180° − (130° + 35° 36′ 17″) = 14° 23′ 43″.

EXERCISE 43

1. X = 2.426(sine 30°) = 2.426 × .50000 = 1.213.

2. X = 8(sine 69°) = 8 × .93358 = 7.4686.

3. X = 7.5(tangent 24°) = 7.5 × .44523 = 3.3392.

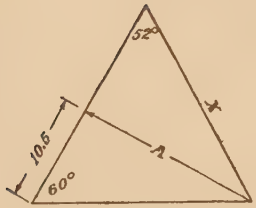


FIG. 375S

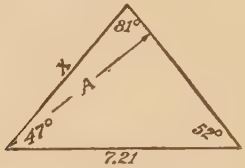


FIG. 376S

4. A = 10.5 (tangent 60°) = 10.5 × 1.7320 = 18.186.
X = 18.186 (cosecant 52°) = 18.186 × 1.2690 = 23.078.

5. A = 7.21 (sine 52°) = 7.21 × .78801 = 5.6815.
X = 5.6815 (cosecant 81°) = 5.6815 × 1.0125 = 5.7525.

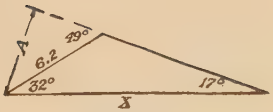


FIG. 377S

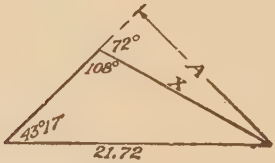


FIG. 378S

6. A = 6.2 (sine 49°) = 6.2 × .75471 = 4.6792.
X = 4.6792 (cosecant 17°) = 4.6792 × 3.4203 = 16.004.

7. A = 21.75 (sine 43° 17′) = 21.75 × .68561 = 14.8914.
X = 14.8914 (cosecant 72°) = 14.8914 × 1.0515 = 15.6583.

8. B = 18° + 31° = 49°
A = 6 (sine 49°) = 6 (.75471) = 4.52826.
X = 4.52826 (cosecant 18°) = 4.52826 (3.2361) = 14.6539.

FIG. 379S

EXERCISE 44

1. $X=\dfrac{9}{\text{cotangent } 48^\circ\ 2'+\text{cotangent } 51^\circ}=\dfrac{9}{.89935+.80978}=5.2658.$

2. $X=\dfrac{4.72}{\text{cotangent } 52^\circ+\text{cotangent } 62^\circ}=3.5948.$

3. $X=\dfrac{6}{\text{cotangent } 21^\circ-\text{cotangent } 50^\circ}=\dfrac{6}{2.6051-.83910}=3.3975.$

4. $Y=\dfrac{10.2^2-6.6^2}{2(11.6)}=2.6069''.$

$A=\dfrac{11.6''}{2}-2.6069''=3.1931''.$

$B=\dfrac{11.6''}{2}+2.6069''=8.4069''.$

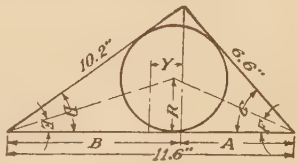


FIG. 385S

Cosine $C=\dfrac{3.1931}{6.6}=.48380$; $C=61^\circ\ 3'\ 58''.$

Cosine $D=\dfrac{8.4069}{10.2}=.82420$; $D=34^\circ\ 29'\ 34''.$

$E=\frac{1}{2}(34^\circ\ 29'\ 34'')=17^\circ\ 14'\ 47''.$

$F=\frac{1}{2}(61^\circ\ 3'\ 58'')=30^\circ\ 31'\ 59''.$

$R=\dfrac{11.6}{\text{cotangent } 17^\circ\ 14'\ 47''+\text{cotangent } 30^\circ\ 31'\ 59''}=\dfrac{11.6}{3.2229+1.6954}=\dfrac{11.6}{4.9183}=2.3585''.$

Diameter $=2(2.3585'')=4.717''.$

5. $A=8''(\text{sine } 59^\circ)=8''(.85717)=6.85736''.$

Sine $B=\dfrac{6.85736}{7.6}=.90228$; $B=64^\circ\ 27'\ 35''.$

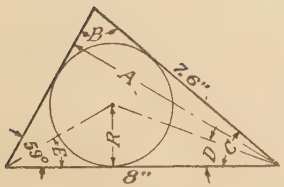


FIG. 386S

$C=180^\circ-(59^\circ+64^\circ\ 27'\ 35'')=56^\circ\ 32'\ 25''.$

$D=28^\circ\ 16'\ 12''.$ $E=29^\circ\ 30'.$

$R=\dfrac{8''}{\text{cot. } 29^\circ\ 30'+\text{cot. } 28^\circ\ 16'\ 12''}=\dfrac{8''}{1.7675+1.8595}=2.2056''.$

Diameter $=2(2.2056'')=4.4102''.$

EXERCISE 45

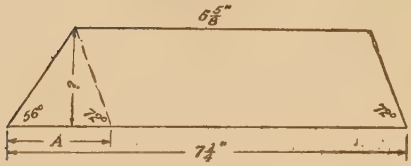


FIG. 387S

1.

$A = 7\frac{1}{4}'' - 5\frac{5}{8}'' = 1\frac{5}{8}''$.

$$? = \frac{1.625''}{\text{cotangent } 56^\circ + \text{cotangent } 72^\circ} = 1.6259''.$$

2. $A = .4375''(\text{cosecant } 14^\circ\ 30') =$
$.4375''(3.9939) = 1.74733''$.
$C = \frac{1}{2}(1\frac{3}{32}'') = .54687''$.
$D = .54687''(\text{cotangent } 14^\circ\ 30'') =$
$.54687''(3.8667) = 2.11458''$.
$E = 1.74733'' + .4375'' = 2.18483''$.
$? = 2.1848'' - 2.11458'' = .07022''$.

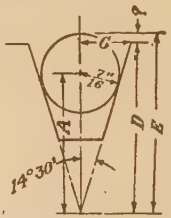


FIG. 388S

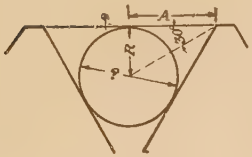


FIG. 389S

3.

$$A = \frac{.250'' - .03125''}{2} = .109375''$$

$R = .1094''(\text{tangent } 30^\circ) =$
$.1094''(.57735) = .06316''$.
$? = 2(.06316'') = .12632''$.

4. $C = .28125''(\text{cotangent } 22^\circ\ 30') =$
$.28125''\ (2.4142) = .67899''$.
$A = 2.500'' - 2(.67899'') +$
$.5625'' = 1.7045''$.

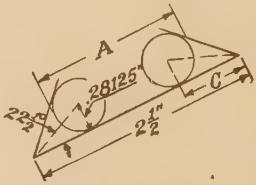


FIG. 390S (a)

FIG. 390S (b)

5. $D = .28125''(\text{cosecant } 45^\circ) =$
$.28125''(1.4142) = .39774''$.
$E = .39774'' + .28125'' = .67899''$.
$B = .750'' - .67899'' = .07101''$.

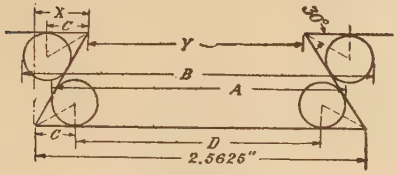


FIG. 391S

6. C = .1875″(cotangent 30°) = .1875″(1.732) = .3247″.
D = 2.5625″ − 2(.3247″) = 1.9131″.
A = 1.9131″ + .375″ = 2.2881″.

7. X = .750(tangent 30°) = .750″(.57735) = .4330″.
Y = 2.5625″ − 2(.433″) = 1.6965″.
B = 1.6965″ + 2(.3247″) + .375″ = 2.7209″.

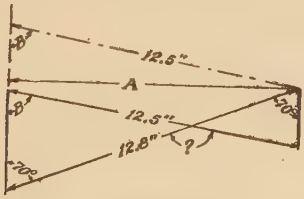


FIG. 392S

8. A = 12.8″(sine 70°) = 12.8″ × .93969 = 12.028″.
Sine $B = \frac{12.028}{12.5} = .96224$.
B = 74° 12′ 15″. ? = 70° + 74° 12′ 15″ = 144° 12′ 15″.

FIG. 393S

9. C = 43″(sine 40°) = 43″(.64279) = 27.63997″.
Sine $A = \frac{27.63997}{32} = .86374$.
A = 59° 44′ 21″.

10. B = 180° − 59° 44′ 21″ = 120° 15′ 39″.

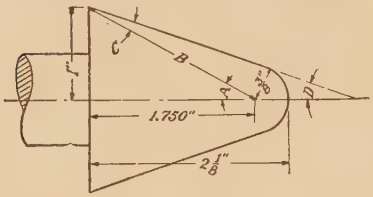


FIG. 394S

11. Cotangent $A = \frac{1.750}{1} = 1.750$. $A = 29° 44' 41''$.

$B = 1''(\text{cosecant } 29° 44' 41'') = 2.0156''$.

Cosecant $C = \frac{2.0156}{.375} = 5.3749$. $C = 10° 43' 20''$.

$D = 29° 44' 41'' - 10° 43' 20'' = 19° 1' 21''$.

$? = 2(19° 1' 21'') = 38° 2' 42''$.

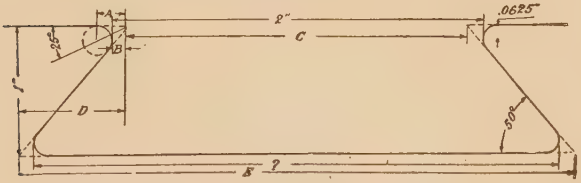


FIG. 395S

12. $A = .0625''(\text{cotangent } 25°) = .0625''(2.1445) = .13403''$.

$B = .13403'' - .0625'' = .07153''$.

$C = 2'' - 2(.07153'') = 1.857''$.

$D = 1''(\text{cotangent } 50°) = .83910''$.

$E = 1.857'' + 2(.83910'') = 3.5352''$.

$? = 3.5352'' - 2(.07153'') = 3.3921''$.

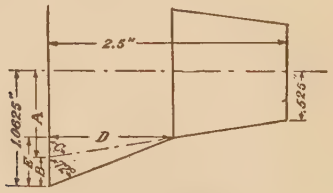


FIG. 396S

13. $A = \frac{1}{2}\left(2.25'' \times \frac{2.5}{12}\right) + .525'' = .75937''$.

$B = 1.0625'' - .75937'' = .30313''$.

$\text{Cotangent } C = \dfrac{\frac{1}{2}(2.25)}{12} = .09375; \quad C = 84° 38' 39''$.

$D = \dfrac{.30313''}{\text{cotangent } 72° - \text{cotangent } 84° 38' 39''} =$

$\dfrac{.30313''}{.32492 - .09375} = \dfrac{.30313''}{.23117} = 1.3113''$.

$E = 1.3113''(\text{cotangent } 72°) = 1.3113''(.32492) = .42607''$.

$? = 2.125'' - 2(.42607'') = 1.27286''$.

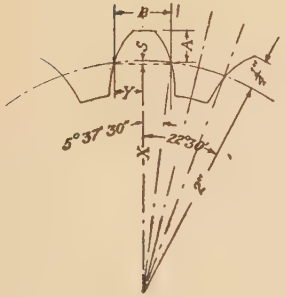


FIG. 397S

14. $X = 2''(\text{cosine } 5° 37' 30'') =$
$2''(.99518) = 1.99036''$.
$S = 2'' - 1.99036'' =$
$.00964''$.
$A = .250'' + .00964'' =$
$.2596''$.

15. $Y = 2''(\text{sine } 5° 37' 30'') =$
$2''(.09801) = .19602''$.
$B = 2Y = 2(.19602'') =$
$.39204''$.

16. $\text{Cotangent } E = \dfrac{14.5}{13} = 1.11538$.

$E = 41° 52' 40''$.
$F = 90° - 41° 52' 40'' =$
$48° 7' 20''$.
$A = 13''(\text{cosecant } 41° 52' 40'') =$
$13''(1.49804) = 19.47452''$.

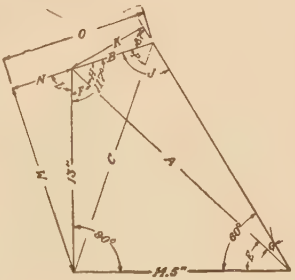


FIG. 398S

17. $G = 60° - 41° 52' 40'' =$
$18° 7' 20''$.
$H = 117° - 48° 7' 20'' =$
$68° 52' 40''$.
$J = 360° - (60° + 117° + 90°) = 93°$.
$K = 19.4745''(\text{sine } 18° 7' 20'') = 19.4745''\ (.311043) = 6.05741''$.
$B = 6.05741''(\text{cosecant } 87°) = 6.05741''\ (1.0014) = 6.06589''$.

18. $L = 180° - 117° = 63°$.
$M = 13''(\text{sine } 63°) = 13''(.89101) = 11.58313''$.
$N = 13''(\text{cosine } 63°) = 13''(.45399) = 5.90187''$.
$O = 6.06589'' + 5.90187'' = 11.96776''$.

$$\text{Cotangent } P = \frac{11.96776}{11.58313} = 1.03321.$$

$P = 44°\ 3'\ 11''$.
$C = 11.58313''(\text{cosecant } 44°\ 3'\ 11'')$.
$C = 11.58313''(1.4381) = 16.656''$.

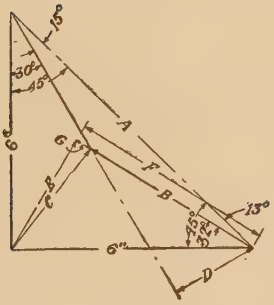


Fig. 399S

19. $A = 6''(\text{secant } 45°) = 6''(1.4142) = 8.4852''$.

20. $D = 8.4852''(\text{sine } 15°) = 8.4852''(.25882) = 2.19614''$.
$B = 2.19614''(\text{cosecant } 28°) = 2.19614''(2.13) = 4.6778''$.

21. $E = 6''(\text{sine } 32°) = 6''(.52992) = 3.17952''$.
$F = 6''(\text{cosine } 32°) = 6''(.84805) = 5.08830''$.

$G = 5.08830'' - 4.6778'' = .4105''$.
$C = \sqrt{3.17952^2 + .4105^2} = 3.2059''$.

22. $D = .125''(\text{sine } 20°) = .125''(.34202) = .04275''$.
$A = 1.500'' + .04275'' = 1.54275''$.

23. $E = .125''(\text{cosine } 20°) = .125''(.93969) = .11746''$.

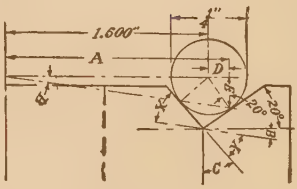


Fig. 400AS

$$\text{Cotangent } B = \frac{1.54275}{.11746} = 13.1342.$$

$B = 4°\ 21'\ 14''$.

24. $X = 20° + 4°\ 21'\ 14'' = 24°\ 21'\ 14''$.
$Y = 24°\ 21'\ 14''$.
$C = 90° - 24°\ 21'\ 14'' - 4°\ 21'\ 14'' = 61°\ 17'\ 32''$.

EXERCISE 46

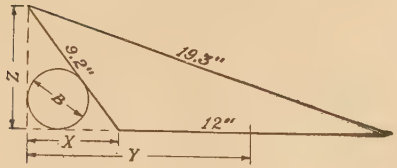


FIG. 401S

1. $Y=\dfrac{19.3^2-9.2^2}{2(12)}=11.9937''.$

   $A=6''+11.9937''=17.9937''.$

2. $X=11.9937''-6''=5.9937''.$

   $Z=\sqrt{9.2^2-5.9937^2}=6.9796''.$

   $B=6.9796''+5.9937''-9.2''=3.7733''.$

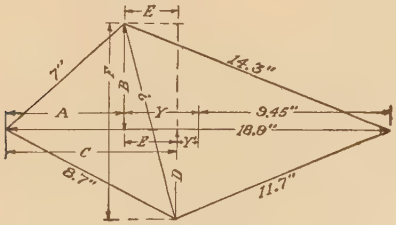


FIG. 402S

3. $Y=\dfrac{14.3^2-7^2}{2(18.9)}=4.1135''.$

   $Y'=\dfrac{11.7^2-8.7^2}{2(18.9)}=1.6190''.$

   $A=9.45''-4.1135''=5.3365''.$

   $B=\sqrt{7^2-5.3365^2}=4.5301''.$

   $C=9.45''-1.619''=7.8310''.$

   $D=\sqrt{8.7^2-7.8310^2}=3.7902''.$

   $E=Y-Y'=4.1135''-1.619''=2.4945''.$

   $F=B+D=4.5301''+3.7902''=8.3203''.$

   $?=\sqrt{8.3203^2+2.4945^2}=8.6862''.$

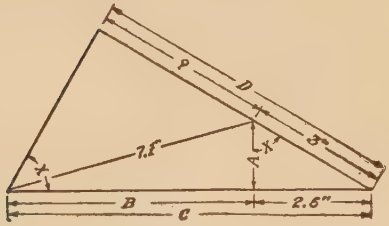


FIG. 403S

4. $A=\sqrt{3^2-2.5^2}=1.6583''$.

$B=\sqrt{7.1^2-1.6583^2}=6.9036''$.

$C=2.5''+6.9036''=9.4036''$.

$D=9.4036''\times\frac{2.5}{3}=7.8363''$.

$?=7.8363''-3''=4.8363''$.

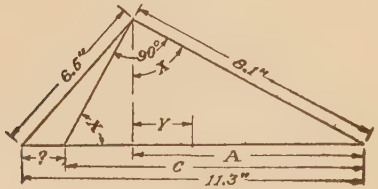


FIG. 404S

5. $Y=\frac{8.1^2-6.5^2}{2(11.3)}=1.0336''$.

$A=\frac{1}{2}(11.3'')+1.0336''=6.6836''$.

$C=8.1''\times\frac{8.1}{6.6836}=9.8165''$.

$?=11.3''-9.8165''=1.4835''$.

EXERCISE 47

1. $X = 1.5''(\text{cosine } 30°) = 1.5''(.86603) = 1.29904''$.
$R = \frac{1}{2}(1.1875'') = .59375''$.
$E = .59375''(\text{cosecant } 45°) =$
$.59375''(1.4142) = .83968''$.
$C = 30° + 45° = 75°$.
$D = .83968''(\text{sine } 75°) =$
$.83968''(.96592) =$
$.811064''$
$A = .59375'' + .811064'' -$
$1.29904'' = .10577''$.

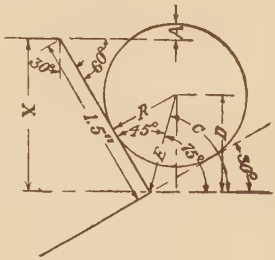


Fig. 405S

2. $B = 2(5)(\text{sine } 15°) =$
$10(.25882) = 2.5882''$.

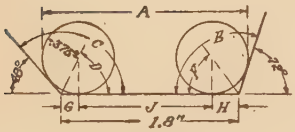


Fig. 406S

3. $C = 180° - 48° = 132°$.
$D = \frac{1}{2}(132°) = 66°$.
$E = 180° - 72° = 108°$.
$F = \frac{1}{2}(108°) = 54°$.
$G = .375''(\text{cotangent } 66°) =$
$.375''(.44523) = .16696''$.
$H = .375''(\text{cotangent } 54°) = .375''(.72654) = .27245''$.
$J = 1.8'' - (.16696'' + .27245'') = 1.3606''$.
$A = 1.3606'' + .750'' = 2.1106''$.

4. $B = 5''(\text{sine } 48°) = 5''(.74314) = 3.7157''$.

5. $C = \dfrac{180° - 12\frac{1}{2}°}{2} = 83°\ 45'$.
$D = \frac{1}{2}(83°\ 45') = 41°\ 52'\ 30''$.
$R = .750''(\text{tangent } 41°\ 52'\ 30'') =$
$.750''(.89646) = .67234''$.
$A = 2(.67234'') = 1.34468''$.

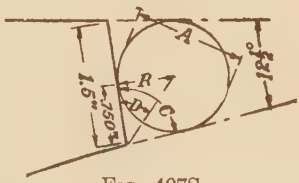


Fig. 407S

6. $B = 10''(\text{sine } 12\frac{1}{2}°) = 10''(.21644) = 2.1644''$.

7. Tangent of $\frac{1}{2}C = \frac{\frac{1}{2}(4.50)}{12} = .1875.$

$\frac{1}{2}C = 10° 37' 10''.$ $C = 21° 14' 20''.$

$A = 10''(\text{sine } 21° 14' 20'') = 10''(.36226) = 3.6226''$

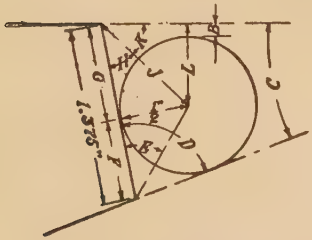


FIG. 408S

8. $D = 90° - 10° 37' 10'' = 79° 22' 50''.$

$E = 39° 41' 25''.$

$F = .5''(\text{cotangent } 39° 41' 25'') = .5''(1.2049) = .60245''.$

$G = 1.375'' - .60245'' = .77255''.$

$\text{Cotangent } H = \frac{.77255}{.500} = 1.5451.$

$H = 32° 54' 42''.$

$J = .5''(\text{cosecant } 32° 54' 42'') = .5''(1.8405) = .92025''.$

$K = 79° 22' 50'' - 32° 54' 42'' = 46° 28' 8''.$

$L = .92025''(\text{sine } 46° 28' 8'') = .92025''(.725) = .66718''.$

$B = .66718'' - .5000'' = .16718''.$

EXERCISE 48

1. $C = 1.18''(\text{secant } 26°) = 1.18''(1.1126) = 1.312868''.$

$D = 90° + 22° - 26° = 86°.$

$E = \frac{1}{2}(86°) = 43°.$

$F = 90° + 26° = 116°$

$G = \frac{1}{2}(116°) = 58°.$

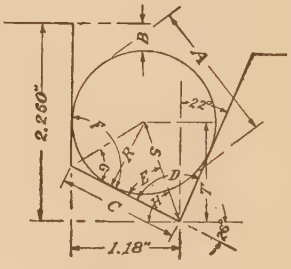


FIG. 409S

$R = \frac{1.312868''}{\text{cotangent } 43° + \text{cotangent } 58°} = \frac{1.312868''}{1.0724 + .62487} = .77351''.$

$H = E + 26° = 69°.$

$A = 2(.77351'') = 1.5470''.$

2. $S = .77351''(\text{cosecant } 43°) = .77351''(1.4663) = 1.1341''.$

$T = 1.1341''(\text{sine } 69°) = 1.1341''(.93358) = 1.05877''.$

$B = 2.250'' - (1.05877'' + .77351'') = .41773''.$

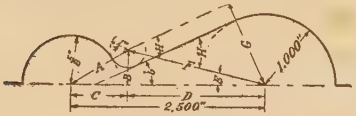


FIG. 410S

3. $A = \frac{5}{8}'' + \frac{1}{4}'' = .875''$. $B = \frac{5}{16}'' + \frac{1}{4}'' = .5625''$.

$C = \sqrt{.875^2 - .5625^2} = .67023''$. $D = 2.5'' - .67023'' = 1.8298''$.

$\text{Cotangent } E = \frac{1.8298}{.5325} = 3.2529$. $E = 17° 5' 18''$.

$F = .5625''(\text{cosecant } 17° 5' 18'') = .5625''(3.4031) = 1.9142''$.

$G = 1.000'' + .250'' = 1.25''$.

$\text{Cosecant } H = \frac{1.9142}{1.25} = 1.53136$. $H = 40° 46' 5''$.

$? = H - E = 40° 46' 5'' - 17° 5' 18'' = 23° 40' 47''$.

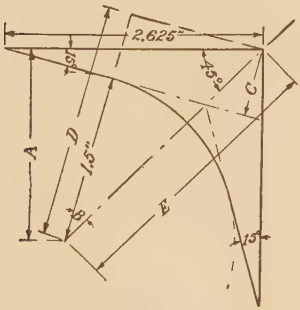


FIG. 411S

4. $C = 2.625''(\text{sine } 15°) = 2.625''(.25882) = .67940''$.

$D = .67940'' + 1.5'' = 2.1794''$. $B = 45° - 15° = 30°$.

$E = 2.1794''(\text{secant } 30°) = 2.1794''(1.1547) = 2.51655''$.

$A = 2.51655''(\text{sine } 45°) = 2.51655''(.70711) = 1.7795''$.

5. $Y=\dfrac{9.5^2-6.000^2}{2(14.500)}=1.8707''.$

$A=\frac{1}{2}(14.5'')-1.8707''=5.3793''.$

$B=\sqrt{6^2-5.3793^2}=2.65767''.$

$\text{Diameter}=6.00''\times\dfrac{9.5}{2.65767}=21.4473''.$

$?=\frac{1}{2}(21.4473'')=10.7236''.$

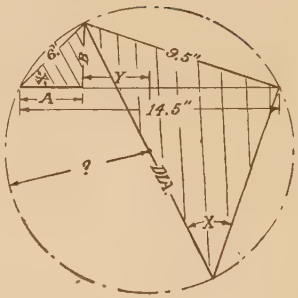


Fig. 412S

NOTE:—The two right triangles are similar.

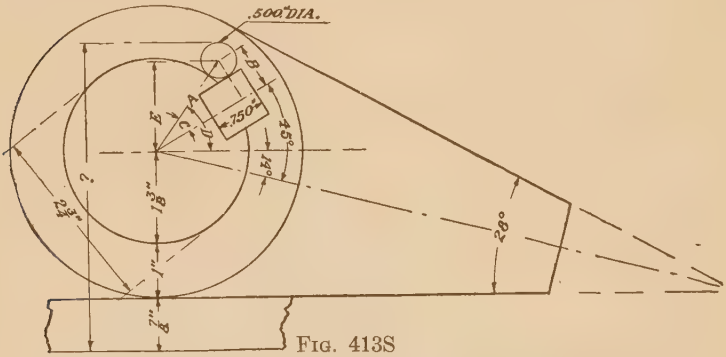


Fig. 413S

6. $A=\frac{1}{2}(2.750''+.500'')=1.625''.$ $B=\frac{1}{2}(.500''+.750'')=.625''.$

$\text{Cosecant } C=\dfrac{1.625}{.625}=2.600.$ $C=22°\ 37'\ 10''.$

$D=22°\ 37'\ 10''+45°-14°=53°\ 37'\ 10''.$

$E=A(\text{sine } 53°\ 37'\ 10'')=1.625''(.80510)=1.3083''.$

$?=.250''+1.3083''+1.375''+1.00''+.875''=4.8083''.$

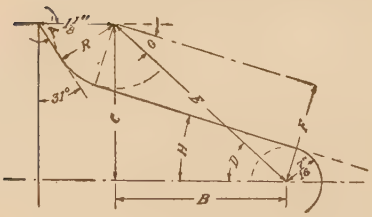


Fig. 414S

7. $A=90°-31°=59°.$

$R=1.125''(\text{sine } 59°)=1.125''(.85717)=.96432''.$

8. $B = 3.25'' - 1.125'' = 2.125''$.
$C = \frac{1}{2}(4.250'') = 2.125''$.
Cotangent $D = \dfrac{2.125}{2.125} = 1$. $D = 45°$.
$E = 2.125''(\text{cosecant } 45°) = 2.125''(1.4142) = 3.0052''$.
$F = .96432'' + .4375'' = 1.40182''$.
Sine $G = \dfrac{1.40182}{3.0052} = .46646$. $G = 27° 48' 17''$.
$H = 45° - 27° 48' 17'' = 17° 11' 43''$.
$? = 2(17° 11' 43'') = 34° 23' 26''$.

EXERCISE 49

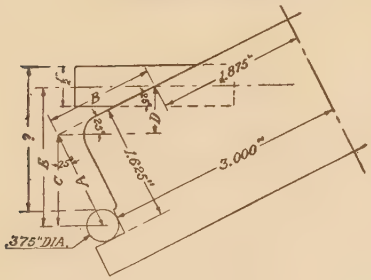


FIG. 415S

1. $A = 1.625'' - \frac{1}{2}(.375'') = 1.4375''$.
$B = 3.000'' + \frac{1}{2}(.375'') - 1.875'' = 1.3125''$.
$C = 1.4375''(\text{cosine } 25°) = 1.4375''(.90631) = 1.30282''$.
$D = 1.3125''(\text{sine } 25°) = 1.3125''(.42262) = .55468''$.
$E = 1.30282'' + .55468'' = 1.8575''$.
$? = 1.8575'' - \frac{1}{2}(.375'') + .250'' = 1.9200''$.

2. $A = 1\frac{1}{4}'' + \frac{1}{2}'' = 1\frac{3}{4}''$.
$B = \frac{1}{4}''$ (tangent 45°) = $.250''$.
$C = 1\frac{3}{4}'' - .250'' = 1.500''$.
$D = 1.500''(\text{sine } 45°) = 1.500''(.70711) = 1.06066''$.
$? = 1.06066'' + .250'' + .200'' = 1.51066''$.

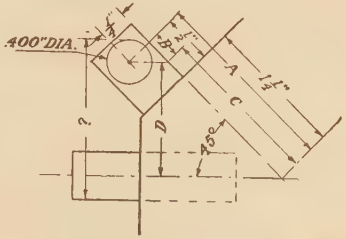


FIG. 416S

EXERCISE 50

1. Outside diameter $= \dfrac{N+2}{D.P.} = \dfrac{22+2}{8} = 3''$.

2. Addendum $= \dfrac{1}{D.P.} = \frac{1}{8} = .125''$.

3. Pitch diameter $= \dfrac{N}{D.P.} = \frac{20}{7} = 2.85714''$.

4. No. $5\frac{1}{2}$ cutter.

5. Whole depth $= \dfrac{2.157}{D.P.} = \dfrac{2.157}{7} = .30814''$.

6. $\frac{1}{2}(\frac{16}{9} + \frac{20}{9}) = 2''$.

7. Clearance $= \dfrac{.157}{9} = .01744''$.

8. If O.D. $= \dfrac{N+2}{D.P.}$; then, D.P. $= \dfrac{N+2}{O.D.} = \dfrac{21+2}{3.2857} = 7$.

9. Chordal thickness $= \dfrac{R}{D.P.} = \dfrac{1.5694}{7} = .2242''$.

10. Depth to set calipers $= \dfrac{U}{D.P.} = \dfrac{1.0294}{7} = .14705''$.

11. $Z = S\left[\text{secant}\left(14\frac{1}{2}° + \dfrac{90°}{N}\right)\right] = .2242''(\text{secant } 18° 47' 9'') =$
$.2242''(1.0562) = .2368''$.

12. $C = \frac{1}{2}P\left(\text{cosine}\dfrac{90°}{N}\right) = \frac{1}{2} \times 3''(\text{cosine } 4° 17' 9'') = 1.5''(.99721) =$
$1.4958''$; or $C = \frac{1}{2}(3.2857'' - .14705'' = 1.4958.''$

$\dfrac{Z}{2} = \dfrac{.2368''}{2} = 1184''$.

$\dfrac{S}{2} = \dfrac{.2242}{2} = .1121''$.

$T = C + \dfrac{Z}{2} + \dfrac{S}{2}\left[\text{tangent}\left(14\frac{1}{2}° + \dfrac{90°}{N}\right)\right] =$
$1.4958'' + .1184'' + .1121''(\text{tangent } 18° 47' 9'') =$
$1.4958'' + .1184'' + .1121''(.33949) = 1.6522.''$

13. $A=2\left(\frac{3.1416}{5}\right)-\frac{3.1416}{4}=.4712''$.

14. $B=3\left(\frac{3.1416}{5}\right)-\frac{3.1416}{2}=.3142''$.

15. $C=\frac{1}{2}\times\frac{1\,9}{8}=1.1875''$.
$D=\frac{1}{2}\times\frac{1\,7}{8}=1.0625''$.
$E=\frac{1}{2}\times\frac{1\,2}{8}=.750''$.
$F=C+D=1.1875''+1.0625''=2.25''$.
$G=E+C=.750''+1.1875''=1.9375''$.
$H=2.25''(\text{sine } 65°)=2.0392''$.
$J=2.25''(\text{cosine } 65°)=.95089''$.
$K=1.9375''-.95089''=.98661''$.
$A=\sqrt{2.0392^2-.98661^2}=2.2653''$.

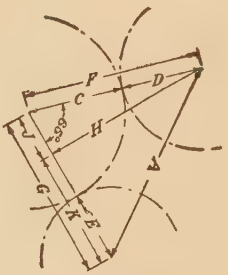


FIG. 428S

16. See Article 225. $A=.1840''$, $B=.1012''$.

EXERCISE 51

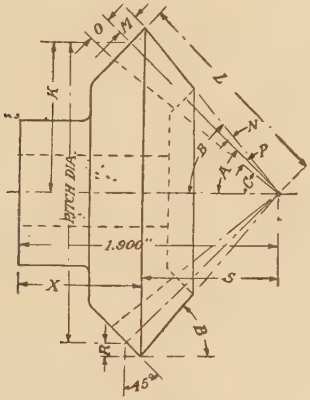


FIG. 432S

1. $A=45°$.

2. Pitch diameter $=\frac{1\,5}{6}=2.5''$.
$K=\frac{1}{2}(2.5'')=1.25''$.
$L=K(\text{cosecant } 45°)=1.25''(1.4142)=1.7677''$.

$M = \dfrac{1}{\text{D.P.}} = \frac{1}{6}''$.

Cotangent $N = \dfrac{1.7677}{\frac{1}{6}} = 10.6062$. $N = 5° 23' 11''$.

$B = 45° + 5° 23' 11'' = 50° 23' 11''$.

3. $O = \dfrac{1.157}{\text{D.P.}} = \dfrac{1.157}{6} = .19283''$.

Cotangent $P = \dfrac{1.7677}{.19283} = 9.1671$. $P = 6° 13' 32''$.

$C = 45° - 6° 13' 32'' = 38° 46' 28''$.

4. $R = \frac{1}{6}''(\text{cosine } 45°) = \frac{1}{6}'' \times .70711 = .1178''$.
Outside diameter = pitch diameter + 2(.1178″) = 2.5″ + .2356″ = 2.7356″.

5. $S = \frac{1}{2}$(outside diameter)(cotangent 50° 23′ 11″) = 1.3678″(.82767) = 1.1321″.
$X = 1.900'' - 1.1321'' = .7679''$.

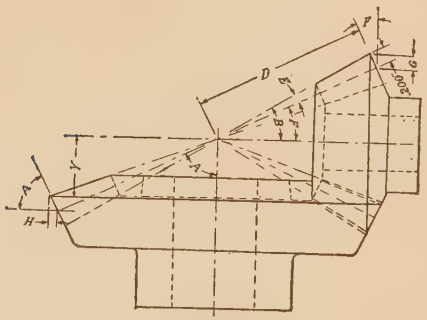


Fig. 433S

6. Pitch diameter of gear $= \frac{2}{5}^{8} = 5.6''$.
Pitch diameter of pinion $= \frac{1}{5}^{3} = 2.6''$.
Tangent of pitch angle $A = \frac{2}{1}^{8}_{3} = 2.1538$.
Angle A = 65° 5′ 41″.

7. $F = 90° - 65° 5' 41'' = 24° 54' 19''$.
D = ½(pitch diameter of pinion)(cosecant 24° 54′ 19″) = 1.3″(2.3746) = 3.087″.

Cotangent $E = \frac{3.087}{.200} = 15.435$. $E = 3° 42' 24''$.

B = face angle of pinion $= 24° 54' 19'' + 3° 42' 24'' = 28° 36' 43''$.

8. $G = .200''(\text{cosine } 24° 54' 19'') = .200''(.90700) = .1814''$.
O.D. of pinion = pitch diameter $+ 2(.1814'') = 2.6'' + .3628'' = 2.9628''$.

9. $H = .200''(\text{cosine } 65° 5' 41'') = .200''(.42118) = .08429''$.
O.D. of gear = pitch diameter $+ 2(.08429'') = 5.6'' + .1686'' = 5.7686''$.

10. $Y = \frac{1}{2}$(O.D. of gear)(cotangent $68° 48' 5'') = 2.8843''(.38785) = 1.1187''$.

EXERCISE 52

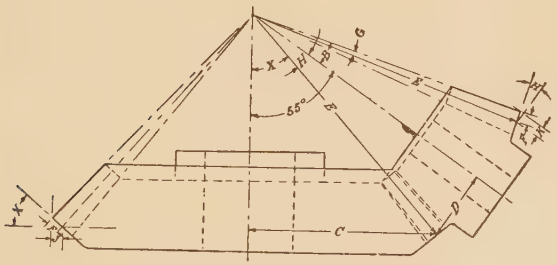


Fig. 435S

1. $C = \frac{1}{2}$(pitch diameter of gear) $= \frac{1}{2}(40)(.5'')\left(\frac{1}{\pi}\right) = 3.1831''$.
$D = \frac{1}{2}$(pitch diameter of pinion) $= \frac{1}{2}(15)(.5'')\left(\frac{1}{\pi}\right) = 1.1936''$.

$$\text{Cotangent } X = \frac{[3.1831'' + 1.1936''(\text{secant } 55°)](\text{cotangent } 55°)}{3.1831}$$

$$= \frac{3.6859}{3.1831} = 1.1580 \qquad X = 40° 48' 43''.$$

2. $E = C(\text{cosecant } 40° 48' 43'') = 3.1831''(1.5300) = 4.8701''$.
$F = .3183(\text{C.P.}) = .3183(.5'') = .15915''$.

$$\text{Cotangent } G = \frac{4.8701}{.15915} = 30.601. \qquad G = 1° 52' 22''.$$

$H = 55° - 40° 48' 43'' = 14° 11' 17''$.
$B = 14° 11' 17'' + 1° 52' 22'' = 16° 3' 39''$.

3. J = .15915″(cosine 40° 48′ 43″) = .15915″(.75686) = .12045″.
O.D. of gear = 2(3.1831″ + .12045″) = 6.6071″.

4. K = .15915″(cosine 14° 11′ 17″) = .15915″(.96950) = .1543″.
O.D. of pinion = 2(1.1936″ + .1543″) = 2.6958″.

EXERCISE 53

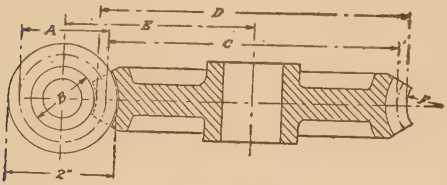


FIG. 441S

1. A = 2 − (.6366)(.5″) = 1.6817″.

2. B = 2 − 2(.6866)(.5″) = 1.3134″.

3. C = (.5″)(45)(.31831) = 7.1619″.

4. D = 7.1619″ + (.6366)(.5″) = 7.4802″.

5. E = $\frac{1}{2}$(7.1619″ + 1.6817″) = 4.4218″.

6. F = $\frac{2″ - 1.2732(.5″)}{2}$ = .6817″.

7. Let X = pitch.
Then A = 3″ − .6366X.
C = X(40)(.31831) = 12.7324X.
Center distance = $\frac{1}{2}$(A + C) = $\frac{1}{2}$(12.7324X + 3″ − .6366X) = $\frac{1}{2}$(3″ + 12.0958X).
Center distance also = 5.5″; therefore,
$\frac{1}{2}$(3″ + 12.0958X) = 5.5″
3″ + 12.0958X = 11″.
12.0958X = 8″.
X = $\frac{8″}{12.0958}$ = .66139″.

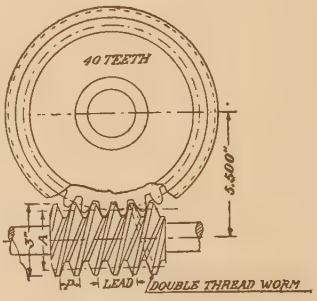


FIG. 442S

8. Lead = 2(.66139″) = 1.32278″.

9. $A=3''-.66139''(.6366)=3''-.4210''=2.5790''$.

EXERCISE 54

1. 40 revolutions.

2. $\frac{3}{7}\times 40=\frac{120}{7}=17\frac{1}{7}$ revolutions.

3. $\frac{1}{9}\times 40=\frac{40}{9}=4\frac{4}{9}$ revolutions.

4. $\frac{1}{17}\times 40=2\frac{6}{17}$ revolutions.

5. A must make $3\frac{1}{13}$ revolutions to cause C to make $\frac{1}{13}$ of a revolution. To take care of the fractional part of a revolution of A, the plate must be divisible by 13; therefore, a plate with 39 holes is used.

6. $\frac{1}{13}=\frac{3}{39}$; therefore, A would be moved 3 holes in the 39 circle for $\frac{1}{13}$ of a revolution of the crank.

7. $\frac{1}{206}\times 40=\frac{20}{1030}$ revolution.

8. If 1 revolution of A causes C to revolve 9°, to cause C to revolve 22°, the crank A must make $22\div 9$ or $2\frac{4}{9}$ revolutions.

9. $82\frac{1}{2}$ revolutions.

10. If 1 revolution of A causes C to revolve 9°, to cause C to revolve 21° 27′ 21″, the crank A must make $\frac{21^\circ\,27'\,21''}{9^\circ}$ or $\frac{77241''}{32400''}$ or $2\frac{4147}{108800}$ revolutions.

11. While C makes $\frac{1}{107}$ of a revolution, A makes $\frac{40}{107}$ of a revolution. But $\frac{40}{107}$ equals approximately $\frac{10}{27}$. If A is revolved $\frac{10}{27}$ of a revolution, it would be off the difference between $\frac{40}{107}$ and $\frac{10}{27}$ or $\frac{10}{27\times 107}$ of a revolution. Thus, $\frac{40\times 27}{107\times 27}-\frac{10\times 107}{27\times 107}=\frac{10}{27\times 107}$. While D makes $\frac{1}{107}$ of a revolution, the plate B and gear E must make $\frac{10}{27\times 107}$ revolutions. By inverting these revolutions the number of teeth in the gears D and E are obtained. Thus, $D=\frac{10}{27\times 107}$ of a tooth and $E=\frac{1}{107}$ of a tooth. By multiplying both of these by 107×27 to clear of fractions, D = 10 teeth and E = 27 teeth. A better set of gears would be D = 30 teeth and E = 81 teeth. Move the crank 10 holes in the 27 circle.

12. 123 divisions solved by the formula, $\frac{D}{E}=40-(X)(N)$.

$X=\frac{40}{N}=\frac{40}{123}=$ approximately $\frac{10}{31}$.

$\frac{D}{E}=40-\frac{10}{31}\times 123=40-\frac{1230}{31}=40-39\frac{21}{31}=\frac{10}{31}=\frac{30}{93}$.

Move the crank 10 holes in the 31 circle; D = 30 teeth; E = 93 teeth.

13. Move the crank 3 holes in the 39 circle; D = 28 teeth; E = 26 teeth.

14. Move the crank 1 hole in the 29 circle; D = 27 teeth; E = 87 teeth.

EXERCISE 55

1. $\frac{77}{244}$ *Ans.*

			3	5	1	12		
1	0	1	5	6	77			
0	1	3	16	19	244			
			3	5	1	12		

2. $\frac{1}{0}, \frac{0}{1}, \frac{1}{10}, \frac{4}{41}, \frac{21}{215}, \frac{46}{471}$.

EXERCISE 56

1. $\sqrt{2}=1.4142=\frac{14142}{10000}$.

10000	14142	1
8284	10000	2
1716	4142	2
1420	3432	2
296	710	2
236	592	2
	118	

			1	2	2	2	2	2
1	0	1	2	5	12	29	70	
0	1	1	3	7	17	41	99	
			1	2	2	2	2	

Ans. $\frac{99}{70}=1.41428$.

.0008 = Variation.

2. 9063 | 7420 | 1
7420 | 6572 | 4
1643 | 848 | 1
848 | 795 | 1
795 | 53 | 15
795 |
0 |

		1	4	1	1	15	8
1	0	1	4	5	9	77	
0	1	1	5	6	11	94	
		1	4	1	1	15	8

$\frac{9063}{7420}=1.2214$.

Ans. $\frac{94}{77}=1.2208$

.0006 = Variation.

3. $.8143 = \frac{8143}{10000}$

8143	10000	1
7428	8143	4
715	1857	2
427	1430	1
288	427	1
278	288	2
	139	

		1	4	2	1	1	2
1	0	1	4	9	13	22	57
0	1	1	5	11	16	27	70
		1	4	2	1	1	2

$\frac{57}{70} = .81428$ *Ans.*

.00002 = Variation.

EXERCISE 57

1. While A makes 9 revolutions, B makes 10.675 revolutions. Therefore, A = 10.675 teeth and B = 9 teeth. By multiplying these by 1000 to clear of fractions, A = 10675 teeth and B = 9000 teeth. After being reduced by the principle of continued fractions, A = 51 teeth and B = 43 teeth. Thus,

9000	10675	1
8375	9000	5
625	1675	2
425	1250	1
200	425	2
200	400	8
	25	

		1	5	2	1	2
1	0	1	5	11	16	43
0	1	1	6	13	19	51
		1	5	2	1	2

When A = 51 teeth and B = 43 teeth, B will make $9 \times \frac{5}{4}\frac{1}{3}$ or 10.6744 revolutions while A makes 9 revolutions. .0006 = Variation.

2. While A makes 7.45 revolutions, B will make 7.99 revolutions; therefore, A = 7.99 teeth and B = 7.45 teeth. By multiplying these by 100 to clear of fractions, A = 799 teeth and B = 745 teeth. After being reduced by the principle of continued fractions, A = 74 teeth and B = 69 teeth. Thus,

799	745	1
745	702	13
54	43	1
43	33	3
11	10	1
10	10	10
1		

		1	13	1	3	1
1	0	1	13	14	55	69
0	1	1	14	15	59	74
		1	13	1	3	1

When A = 74 teeth and B = 69 teeth, B will make $7.45 \times \frac{7}{6}\frac{4}{9}$ or 7.9898 revolutions while A makes 7.45 revolutions. .0002 = Variation.

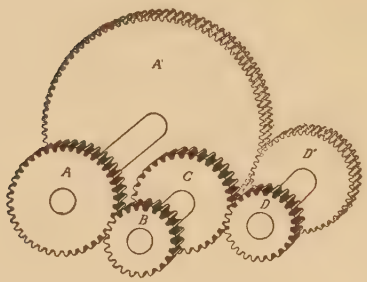
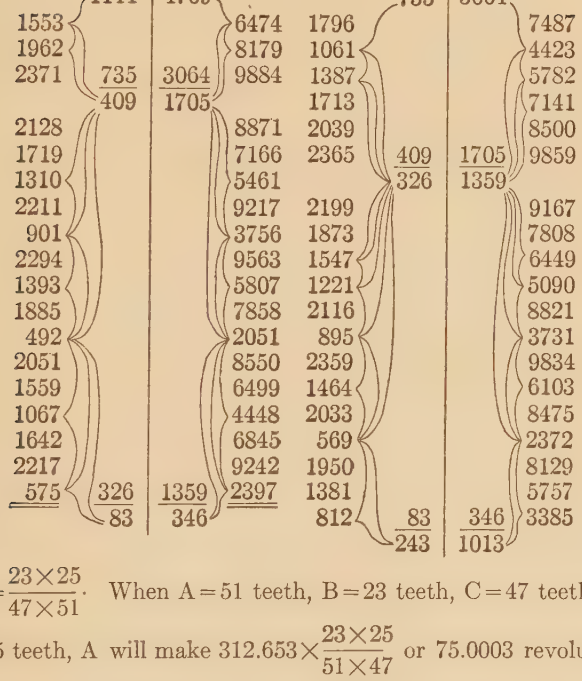


FIG. 449S

3. While the imaginary simple gear A′ makes 75 revolutions, the imaginary simple gear D′ makes 312.653 revolutions; therefore, A′ = 312.653 teeth and D′ = 75 teeth. By multiplying these by 1000 to clear of fractions, A′ = 312653 and D′ = 75000 teeth. After being reduced by the principle of continued fractions, A′ = 2397 teeth and B′ = 575 teeth. Thus,

312653	75000	4
300000	63265	5
12653	11735	1
11735	11016	12
918	719	1
719	597	3
199	122	1
122	77	1
77	45	1
45	32	1
32	13	2
26	12	2
6	1	6

												2 0
		4	5	1	12	1	3	1	1	1	1	1144
1	0	1	5	6	77	83	326	409	735	1144	1879	4769
0	1	4	21	25	321	346	1359	1705	3064	4769	7833	735
		4	5	1	12	1	3	1	1	1	1	3064
												2 1



$\frac{575}{2397}=\frac{23\times25}{47\times51}$. When A = 51 teeth, B = 23 teeth, C = 47 teeth, and D = 25 teeth, A will make $312.653\times\frac{23\times25}{51\times47}$ or 75.0003 revolutions, while D makes 312.653 revolutions. $\frac{575}{2397}$ is the first set that can be factored; therefore it is the closest set, .0003 = Variation. Many other sets may be obtained by simply uniting some of the smaller convergents, but the variation will be greater.

Thus, $\frac{77+83}{321+346}=\frac{160}{667}=\frac{10\times16}{23\times29}$.

$$312.653\times\frac{10\times16}{23\times29}=74.9985. \quad .0015=\text{Variation.}$$

EXERCISE 58

1. .1763″.

2. $.4311\div\frac{1}{5}$ or 2.1555 revolutions.

3. $.1027\div\frac{1}{3\frac{1}{2}}$ or 1.027 revolutions.

4. While the spindle makes 1 revolution, the lead screw makes .5431 revolutions. By inverting these revolutions to obtain the number of teeth in the gears to be used, the gear on the spindle = .5431 of a tooth, and the gear on the lead screw = 1 tooth. By multiplying these by 10000 to clear of fractions, the gear on the spindle = .5431 × 10000 or 5431 teeth, and the gear on the lead screw = 1 × 10000 or 10000 teeth. These numbers are then reduced to suitable numbers for gears by the principle of continued fractions.

Thus,

10000	5431	1
5431	4569	1
4569	862	5
4310	777	3
259	85	

		1	1	5	3	2
1	0	1	1	6	19	44
0	1	1	2	11	35	81
		1	1	5	3	2

If the gear on the spindle had 44 teeth, and the gear on the lead screw had 81 teeth, the lead screw would make $\frac{44}{81}$ or .5432 revolution while the spindle made 1 revolution. The variation = .0001″.

5. $\frac{45}{32} \times \frac{1}{5}'' = .28125''$.

6. To move the slide or tool .3153″, the lead screw must make $.3153 \div \frac{1}{6}$ or 1.8918 revolutions; therefore, while the spindle makes 1 revolution, the lead screw must make 1.8918 revolutions. By inverting these revolutions to obtain the number of teeth in the gears to be used, the gear on the spindle = 1.8918 teeth, and the gear on the lead screw = 1 tooth. By multiplying these by 10000 to clear of fractions, the gear on the spindle = 1.8918 × 10000 or 18918 teeth and the gear on the lead screw 1 × 10000 or 10000 teeth. These numbers are then reduced to suitable numbers for gears by the principle of continued fractions. Thus,

10000	18918	1
8918	10000	1
1082	8918	8
1048	8656	4
34	262	

		1	1	8	4
1	0	1	1	9	37
0	1	1	2	17	70
		1	1	8	4

If the gear on the spindle had 70 teeth, and the gear on the lead screw had 37 teeth, while the spindle made 1 revolution, the slide would move $\frac{70}{37} \times \frac{1}{6}''$ or .31531″. Variation = .00001″.

EXERCISE 59

1. While the spindle makes one revolution, the lead screw will make $\frac{50}{70}\times\frac{37}{90}$ or $\frac{37}{126}$ of a revolution. If one revolution of the lead screw moves the carriage $\frac{1}{6}$ of an inch, $\frac{37}{126}$ of a revolution will move it $\frac{37}{126}\times\frac{1}{6}''$ or .04894″. Therefore, the lathe will cut a .04894″ lead.

2. While the spindle makes 1 revolution, the imaginary simple gear D′ makes 1 revolution. While the lead screw makes 4.011 revolutions, the imaginary simple gear G′ makes 4.011 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 1 revolution while G′ makes 4.011 revolutions, D′ must have 4.011 teeth and G′ 1 tooth. By multiplying these by 1000 to clear of fractions, D′ = 4011 teeth and G′ = 1000 teeth. The factors of these numbers are too large for compound gears; therefore, the numbers are reduced by the principle of continued fractions to other numbers from which suitable numbers for gears may be obtained. (See Articles 240 and 246.) Thus,

4011	1000	4
4000	990	90
11	10	1
10	10	10
1	0	∞

						2 ∞
		4	90	1	10	2091
1	0	1	90	91	1000	8387
0	1	4	361	365	4011	1909
		4	90	1	10	7657
						2 ∞

	2091	8387				1909	7657	
	1000	4011				1000	4011	
	1091	4376				909	3646	
2273			9117	1727		91	365	6927
1182			4741			818	3281	
2455			9847	1645				9478
1273	1000	4011	5106	2472		91	365	6197
	91	365				827	2916	

Since $\frac{4011}{1000}$ = approximately $\frac{5106}{1273}$, D′ = 5106 teeth, and G′ = 1273 teeth. After factoring (5106 = 69×74; 1273 = 19×67) and placing in proper compound order, D = 69 teeth, E = 19 teeth, F = 74 teeth, and G = 67 teeth.

CHECK: While the spindle makes one revolution, the lead screw will make $\frac{69\times74}{19\times67}$ or 4.01092 revolutions, which varies only .00008 from the true value.

3. While the spindle or the imaginary simple gear D′ makes 1 revolution, the carriage must move 3.9248″. To move the carriage 3.9248″, the lead screw or the imaginary simple gear G′ must make $3.9248\div\frac{1}{4}$ or 15.6992 revolutions; therefore, while the imaginary simple gear D′ makes 1 revolution, the imaginary simple gear G′ must make 15.6992 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 1 revolution while G′ makes 15.6992 revolutions, D′ must have 15.6992 teeth and G′ 1 tooth. By multiplying these by 10000 to clear of fractions, D′ = 156992 teeth and G′ = 10000 teeth. The factors of these numbers are too large for compound gears; therefore, they are reduced by the principle of continued fractions to other numbers from which suitable factors can be obtained. (See Articles 240 and 246.) Thus,

10000	156992	15
6992	150000	1
3008	6992	2
2928	6016	3
80	976	12
80	960	5
0	16	∞

			15	1	2	3	12	5	∞ 0
									123
1	0	1	1	3	10	123	625	1931	
0	1	15	16	47	157	1931	9812	502	
		15	1	2	3	12	5	7881	
								∞ 1	

						123	1931	
				605				9498
	502	7881		482				7567
	123	1931		359				5636
	379	5950		595				9341
635	123	1931	9969	236				3705
	256	4019		585				9184
389			6107	349				5479
522	123	1931	8195	462				7253
	133	2088		575		10	157	9027
						113	1774	

Since $\frac{156992}{10000}$ = approximately $\frac{2088}{133}$, D′ = 2088 teeth and G′ = 133 teeth. After factoring $\left(\frac{2088}{133}=\frac{87\times24}{7\times19}=\frac{87\times72}{21\times19}\right)$ and placing in

proper compound order, D = 87 teeth, E = 21 teeth, F = 72 teeth, and G = 19 teeth.

CHECK: While the spindle makes 1 revolution, the carriage will move $\frac{87\times72}{21\times19}\times\frac{1''}{4}$ or 3.92481″, which varies .00001″ from the true value.

4. While the spindle or gear D makes 1 revolution, the carriage must move 3.1429″. To move the carriage 3.1429″, the lead screw must make $3.1429\div\frac{1}{4}$ or 12.5716 revolutions. While the lead screw makes 12.5716 revolutions, F and E will make $12.5716\times\frac{24}{96}$ or 3.1429 revolutions; therefore, while the spindle or gear D makes 1 revolution, E must make 3.1429 revolutions.

If we know the number of revolutions the gears D and E must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 71.) Thus, if D makes 1 revolution while E makes 3.1429 revolutions, D must have 3.1429 teeth and E 1 tooth. By multiplying these by 10000 to clear of fractions, D = 31429 teeth and E = 10000 teeth. These numbers are too large; therefore, they are reduced by the principle of continued fractions. Thus,

10000	31429	3
8574	30000	6
1426	1429	1
	1426	
	3	

		3	6	1
1	0	1	6	7
0	1	3	19	22
		3	6	1

Since $\frac{3}{1}\frac{1}{0}\frac{4}{0}\frac{2}{0}\frac{9}{0}$ = approximately $\frac{2}{7}\frac{2}{1}=\frac{6}{2}\frac{6}{1}$, D = 66 teeth and E = 21 teeth.

CHECK: While the spindle makes 1 revolution, the carriage will move $\frac{6}{2}\frac{6}{1}\times\frac{9}{2}\frac{6}{4}\times\frac{1}{4}''$ or 3.14285″, which varies only .00005″ from the true value.

EXERCISE 60

1. While the spindle makes 1 revolution, gear B will make $\frac{48}{32}$ revolutions. Then, 1 revolution of the spindle, or $\frac{48}{32}$ revolutions of gear B, will move the carriage $\frac{1}{7}$ of an inch. If $\frac{48}{32}$ revolutions of gear B moves the carriage $\frac{1}{7}$ of an inch, to move it .1537″, B must make $\frac{48}{32}\times\frac{.1537}{\frac{1}{7}}$ or 1.61385 revolutions. Therefore, while the

spindle, or gear A, makes one revolution, B must make 1.61385 revolutions.

If we know the number of revolutions the gears A and B must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 71.) Thus, if A makes 1 revolution while B makes 1.61385 revolutions, A must have 1.61385 teeth and B 1 tooth. By multiplying these by 100000 to clear of fractions, A = 161385 teeth, and B = 100000 teeth. These numbers are too large; therefore, they are reduced by the principle of continued fractions.

Thus,

100000	161385	1
61385	100000	1
38615	61385	1
22770	38615	1
15845	22770	1
13850	15845	2
1995	6925	3
1880	5985	2
	940	

		1	1	1	1	1	2	3	2	1
1	0	1	1	2	3	5	13	44	57	
0	1	1	2	3	5	8	21	71	92	
		1	1	1	1	1	2	3	2	1

Since $\frac{161385}{100000}$ = approximately $\frac{92}{57}$, A = 92 teeth, and B = 57 teeth.

CHECK: While the spindle makes 1 revolution, B will make $\frac{92}{57}$ revolutions. If $\frac{48}{32}$ revolutions of B moves the carriage $\frac{1}{7}$ of an inch, $\frac{92}{57}$ revolutions of B will move it $\frac{1}{7}'' \times \frac{\frac{92}{57}}{\frac{48}{32}}$ or .153717″, which varies .000017″ from the true value.

2. While the spindle makes one revolution, gear B will make $\frac{48}{32}$ revolutions. Then, 1 revolution of the spindle, or $\frac{48}{32}$ revolutions of B, will move the carriage $\frac{1}{7}$ of an inch. When A = 47 teeth and B = 33 teeth, B will make $\frac{47}{33}$ revolutions while the spindle makes 1 revolution. If $\frac{48}{32}$ revolutions of B moves the carriage $\frac{1}{7}$ of an inch, $\frac{47}{33}$ revolutions will move it $\frac{1}{7}'' \times \frac{\frac{47}{33}}{\frac{48}{32}}$ or .13564″.

EXERCISE 61

1. $40 \times \frac{48}{28} \times \frac{40}{50} = 48$ revolutions.

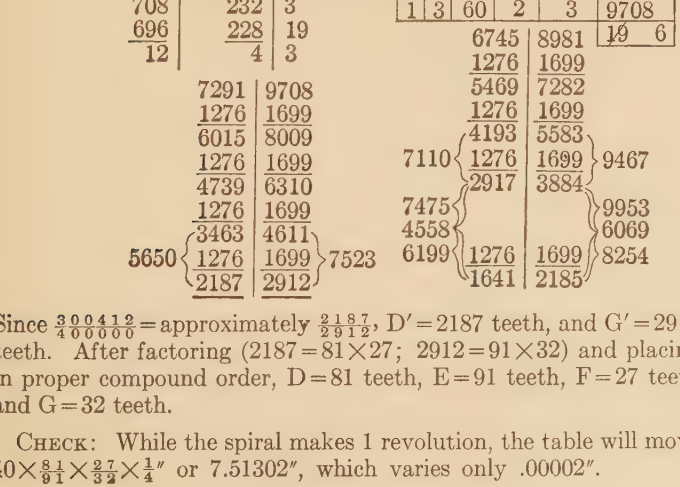
2. $\frac{11.05}{1} \times \frac{1}{4}'' = 2.7625''$.

3. $40 \times \frac{64}{57} \times \frac{62}{36} \times \frac{1}{4}'' = 19.3372''$.

EXERCISE 62

1. While the spiral makes 1 revolution, the table must move 7.5103 inches. To move the table 7.5103 inches the feed screw, or the imaginary simple gear G′, must make $7.5103 \div \frac{1}{4}$ or 30.0412 revolutions; therefore, to cut a 7.5103-inch lead the imaginary simple gear G′ must make 30.0412 revolutions while the spiral makes one revolution. While the imaginary simple gear D′ makes 40 revolutions, the spiral also makes 1 revolution; therefore, while D′ makes 40 revolutions, G′ must make 30.0412 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 40 revolutions while G′ makes 30.0412 revolutions, D′ must have 30.0412 teeth and G′ 40 teeth. By multiplying these by 10000 to clear of fractions, D′ = 300412 teeth, and G′ = 400000 teeth. The factors of these numbers are too large for compound gears; therefore, they are reduced by the principle of continued fractions to other numbers from which suitable factors may be obtained.



Since $\frac{300412}{400000} = \text{approximately } \frac{2187}{2912}$, D′ = 2187 teeth, and G′ = 2912 teeth. After factoring ($2187 = 81 \times 27$; $2912 = 91 \times 32$) and placing in proper compound order, D = 81 teeth, E = 91 teeth, F = 27 teeth and G = 32 teeth.

CHECK: While the spiral makes 1 revolution, the table will move $40 \times \frac{81}{91} \times \frac{27}{32} \times \frac{1}{4}''$ or 7.51302″, which varies only .00002″.

2. While the spiral makes 1 revolution, the table will move 50.0522 inches. To move the table 50.0522 inches the feed screw, or the imaginary simple gear G′, must make $50.0522 \div \frac{1}{4}$ or 200.2088 revolutions; therefore, to cut a 50.0522-inch lead the imaginary simple gear G′ must make 200.2088 revolutions while the spiral makes 1 revolution. While the imaginary simple gear D′ makes 40 revolutions, the spiral also will make 1 revolution; therefore, while D′ makes 40 revolutions, G′ must make 200.2088 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 40 revolutions while G′ makes 200.2088 revolutions, D′ must have 200.2088 teeth and G′ 40 teeth. By multiplying these by 10000 to clear of fractions, D′ = 2002088 teeth, and G′ = 400000 teeth. The factors of these numbers are too large for compound gears; therefore, they are reduced by the principle of continued fractions to other numbers from which suitable factors may be obtained.

Thus,

400000	2002088	5
398808	2000000	191
1192	2088	1
896	1192	1
296	896	3
296	888	37
	8	

							37 1
		5	191	1	1	3	1724
1	0	1	191	192	383	1341	8629
0	1	5	956	961	1917	6712	958
		5	191	1	1	3	4795
							37 1

1533	958	4795	7673
	383	1917	
	575	2878	
1917			9595
1342			6717
767			3839
1726			8639
959			4800
1151			5761
1343			6722
1535			7683
1727			8644
1919			9605
	383	1917	
	192	961	

	1724	8629	
	1341	6712	
	383	1917	
1723			8624
1340			6707
957			4790
1531			7663
574			2873
1913			9575
1339			6702
765			3829
1721			8614
956			4785
1147			5741
1338			6697
1529	192	961	7653
	191	956	

Since $\frac{2002088}{4000000}$ = approximately $\frac{961}{1921}$, D′ = 961 teeth, and G′ = 192 teeth. After factoring and raising the factors $\left(\frac{961}{192}=\frac{31\times31}{12\times16}=\frac{93\times62}{36\times32}\right)$ and placing in proper compound order, D = 93 teeth, E = 36 teeth, F = 62 teeth, and G = 32 teeth.

CHECK: While the spiral makes 1 revolution, the table will move $40\times\frac{93}{36}\times\frac{62}{32}\times\frac{1}{4}''$ or 50.05203″, which varies only .00017″ from the true lead.

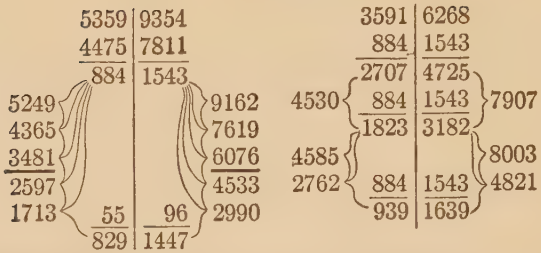
3. While the spiral makes 1 revolution, the table will move 5.7291 inches. To move the table 5.7291 inches the feed screw, or the imaginary simple gear G′, must make $5.7291\div\frac{1}{4}$ or 22.9164 revolutions; therefore, to cut a 5.7291-inch lead the imaginary simple gear G′ must make 22.9164 revolutions while the spiral makes 1 revolution. While the imaginary simple gear D′ makes 40 revolutions, the spiral also makes 1 revolution; therefore, while D′ makes 40 revolutions, G′ must make 22.9164 revolutions.

If we know the number of revolutions the imaginary simple gears D′ and G′ must make, the number of teeth in them are obtained by inverting the revolutions. (See Article 76.) Thus, if D′ makes 40 revolutions while G′ makes 22.9164 revolutions, D′ must have 22.9164 teeth and G′ 40 teeth. By multiplying these by 10000 to clear of fractions, D′ = 229164 teeth, and G′ = 400000 teeth. The factors of these numbers are too large for compound gears; therefore, they are reduced by the principle of continued fractions to other numbers from which suitable factors may be obtained.

Thus,

400000	229164	1
229164	170836	1
170836	58328	2
116656	54180	1
54180	4148	13
53924	4096	16
256	52	4
208	48	1
48	4	12
48		

		1	1	2	1	13	16	4	1	0
1	0	1	1	3	4	55	884	3591	4475	3591
0	1	1	2	5	7	96	1543	6268	7811	6268
		1	1	2	1	13	16	4	1	5359
										9354
										2



Since $\frac{2}{4}\frac{2}{0}\frac{9}{0}\frac{1}{0}\frac{6}{0}\frac{4}{0}$ = approximately $\frac{3}{6}\frac{4}{0}\frac{8}{7}\frac{1}{6}$, D′ = 3481 teeth, and G′ = 6076 teeth. After factoring ($3481 = 59 \times 59$; $6076 = 62 \times 98$) and placing in proper compound order, D = 59 teeth, E = 62 teeth, F = 59 teeth, and G = 98 teeth.

Check: While the spiral makes 1 revolution, the table will move $40 \times \frac{5}{6}\frac{9}{2} \times \frac{5}{9}\frac{9}{8} \times \frac{1}{4}''$ or 5.729099″, which varies only .000001″ from the true lead.

EXERCISE 63

1. $3 + \frac{2(1.157)}{10} = 3.2314$ inches.

2. Cosecant θ = (D.P.)(pitch diameter) = $3 \times 10 = 30$; therefore, $\theta = 1°\ 54'\ 37''$.

Lead of the hob $= \frac{\pi}{\text{D.P.}}(\text{secant } \theta) = \frac{3.1416}{10}(\text{secant } 1°\ 54'\ 37'') =$

$\frac{3.1416}{10} \times 1.0005 = .31432''$.

3. Lead of the gash $= \pi$(pitch diameter)(cotangent θ) = $3.1416 \times 3'' \times 29.9831 = 282.59''$.

EXERCISE 64

1. While the spindle makes 1 revolution, B will make $\frac{2}{4}\frac{6}{8}$ or 2 revolutions, and the carriage will move $\frac{1}{3}$ of an inch. If 2 revolutions of B moves the carriage $\frac{1}{3}$ of an inch, to move it .31432″ it must make $2 \times \frac{.31432}{\frac{1}{3}}$ or 1.8859 revolutions. While the spindle, or A, makes 1 revolution, B must make 1.8859 revolutions to cause the carriage to move .31432″.

If we know the number of revolutions the gears A and B must make, the number of teeth in them is obtained by inverting the

revolutions. (See Article 71.) Thus, if A makes 1 revolution while B makes 1.8859 revolutions, A must have 1.8859 teeth and B 1 tooth. By multiplying these by 10000 to clear of fractions, A = 18859 teeth and B = 10000 teeth. These numbers are too large for gears; therefore, they are reduced to other numbers by the principle of continued fractions.

Thus,

18859	10000	1
10000	8859	1
8859	1141	7
7987	872	1
872	269	3
807		

		1	1	7	1	3
1	0	1	1	8	9	35
0	1	1	2	15	17	66
		1	1	7	1	3

Since $\frac{10000}{18859}$ = approximately $\frac{35}{66}$, A = 66 teeth, and B = 35 teeth.

CHECK: While the spindle makes 1 revolution, B will make $\frac{66}{35}$ revolutions. If 2 revolutions of B moves the carriage $\frac{1}{3}$ of an inch, $\frac{66}{35}$ revolutions will move it $\frac{1}{3}'' \times \frac{\frac{66}{35}}{2}$ or .31428″, which varies only .00004″ from the true value.

2. According to Articles 265 and 266, when the lead of the hob is .31428″, and the lead of the gash is 282.59″, the adjacent teeth along any flute will be out of line $\frac{1}{900 \cdot 16}$ of a complete revolution.

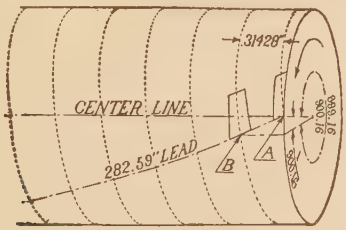


FIG. 462S(a)

Then, when tooth A is on the center line, the hob must make $1 - \frac{1}{900 \cdot 16}$ or $\frac{899 \cdot 16}{900 \cdot 16}$ of a revolution to cause tooth B to come on the center line; therefore, when the hob makes $\frac{899 \cdot 16}{900 \cdot 16}$ of a revolution, the tool must reciprocate 6 times because there are 6 flutes in the hob. If 1 revolution of F causes the tool to reciprocate 4 times,

to cause the tool to reciprocate 6 times, F would have to make $6 \div 4$ or 1.5 revolutions. (See Fig. 462S (b).)

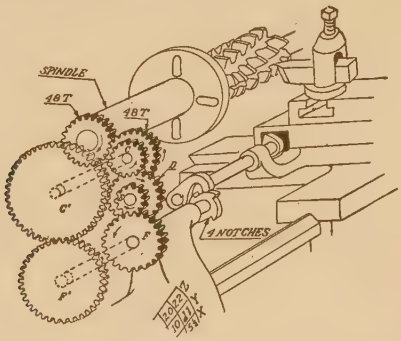


FIG. 462S (b)

If we know the number of revolutions the imaginary simple gears C′ and F′ must make, the number of teeth in them will be obtained by inverting the revolutions. (See Article 76.) Thus, if C′ makes $\frac{899 \cdot 16}{900 \cdot 16}$ of a revolution while F′ makes 1.5 revolutions, C′ must have 1.5 teeth and F′ $\frac{899 \cdot 16}{900 \cdot 16}$ teeth. By multiplying these by 90016 to clear of fractions, $C' = 1.5 \times 90016$ or 135024 teeth and $F' = \frac{899 \cdot 16}{900 \cdot 16} \times 90016$ or 89916 teeth. The factors of these numbers are too large for compound gears; therefore, the numbers are reduced by the principle of continued fractions to other numbers for which suitable factors may be obtained.

Thus,

89916	135024	1
45108	89916	1
44808	45108	1
44700	44808	149
108	300	2
84	216	1
24	84	3
24	72	2
0	12	

								2	1
		1	1	1	149	2	1	3	4196
1	0	1	1	2	299	600	899	3297	6301
0	1	1	2	3	449	901	1350	4951	5695
		1	1	1	149	2	1	3	8552
									2

5095	4196	6301	7651
5994	3297	4951	9001
	899	1350	

	5695	8552	
	3297	4951	
	2398	3601	
6295			9453
3897			5852
5396	899	1350	8103
	1499	2251	

Since $\frac{1350}{899}\frac{24}{16}$ = approximately $\frac{1350}{899}$, C′ = 1350 teeth, and D′ = 899 teeth. After factoring (1350 = 50×27; 899 = 29×31) and placing in proper compound order, C = 50 teeth, D = 29 teeth, E = 27 teeth, and F = 31 teeth.

EXERCISE 65

1. $\text{Pitch diameter} = \frac{\text{N}(\text{secant } \theta)}{\text{D.P.}} = \frac{30(1.1034)}{5} = 6.6204$ inches.

2. $\text{Pitch diameter} = \frac{\text{N}(\text{secant } \theta)}{\text{D.P.}} = \frac{50(1.3054)}{10} = 6.527$ inches.

3. $\text{Lead of the spiral gear} = \text{N}\left(\frac{\pi}{\text{D.P.}}\right)(\text{cosecant } \theta) = \frac{30\times 3.1416}{5}\times 2.3662 = 44.6019$ inches.

4. $\text{Lead of the spiral gear} = \text{N}\left(\frac{\pi}{\text{D.P.}}\right)(\text{cosecant } \theta) = 50\times\frac{3.1416}{5}\times 1.5557 = 24.4364$ inches.

EXERCISE 66

1. See Article 276. There are $20 \div .900$ or $22\frac{2}{9}$ turns of the spiral B within the distance of 1 turn of the spiral A. A straight line will cross $22\frac{2}{9}$ spaces between the turns of the spiral B in the distance equal to 1 turn of the spiral A; but the spiral A making 1 turn in the opposite hand from the spiral B crosses $22\frac{2}{9}+1$ or $23\frac{2}{9}$ spaces. If parallel lines are drawn from the intersecting points of these spirals to the end, they will divide the circle into $23\frac{2}{9}$ parts; therefore, $\text{X} = \frac{1}{23\frac{2}{9}}$ or $\frac{9}{209}$ of a complete circle.

2. $\text{Y} = 1 - \frac{9}{209} = \frac{200}{209}$ of a complete circle.

3. $\text{X} = \frac{1}{22\frac{2}{9}-1} = \frac{1}{21\frac{2}{9}} = \frac{9}{191}$ of a complete circle.

4. $\text{Y} = 1 + \frac{9}{191} = \frac{200}{191}$ of a complete circle.

5. See Article 277. If the part of a complete circle between each of the intersecting points of the spirals is $\frac{1}{12}$, the spiral line A must cross 12 spaces between the turns of the spiral B. When the hand of the spirals is the same, the one spiral crosses the spaces

between the turns of the other spiral once less than a straight line would cross; therefore, a straight line would cross 12+1 or 13 spaces between the turns of the spiral B. If a straight line crosses 13 spaces within the distance of 10 inches, the lead of the spiral B must be $10 \div 13$ or $\frac{10}{13}$ of an inch.

6. If $\frac{201}{200}$ is the relation of the intersecting points, the part of a complete circle between the intersecting points of the spirals must be $\frac{201}{200} - 1$ or $\frac{1}{200}$. The spiral line A crosses 200 spaces between the turns of the spiral B. The hand of the spirals are the same; therefore, a straight line will cross 200+1 or 201 spaces between the turns of the spiral B. Then the lead of the spiral $B = 102 \div 201$ or $\frac{34}{67}$ of an inch.

EXERCISE 67

1. SOLUTION OF THE INDEX GEARS.

Lead of the spiral gear $= N\left(\frac{\pi}{\text{D.P.}}\right)(\text{cosecant } \theta) =$

$$54 \times \frac{3.1416}{3} \times 2.5593 = 144.725''.$$

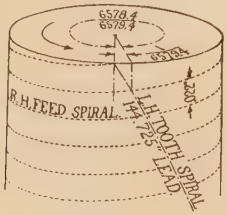


FIG. 486S

The feed spiral will be right-handed because a right-hand hob is being used. (See Article 274.) The hand of the tooth spiral and the hand of the feed spiral are opposite; therefore, according to Articles 275 and 276 the part of a complete circle between the intersecting points is $\frac{1}{6579 \cdot 4}$. If a left-hand hob feeds down through the face of the work, the work must revolve as shown by the large arrow; and from this it can be seen that the work must make $1 - \frac{1}{6579 \cdot 4}$ or $\frac{6578 \cdot 4}{6579 \cdot 4}$ of a revolution, while the hob makes 54 revolutions, since there are 54 teeth in the spiral gear.

While the hob makes 54 revolutions, the imaginary simple gear H′ will make 54. When the work makes $\frac{6578\cdot4}{6579\cdot4}$ of a revolution the imaginary simple gear E′ will make $\frac{6578\cdot4}{6579\cdot4}\times90$ revolutions.

If we know the number of revolutions the simple gears H′ and E′ must make, the number of teeth in them is obtained by inverting the revolutions. (See Article 76.) Thus, if H′ makes 54 revolutions while E′ makes $\frac{6578\cdot4}{6579\cdot4}\times90$ revolutions, H′ must have $\frac{6578\cdot4}{6579\cdot4}\times90$ teeth and E′ = 54 teeth. By multiplying these by $\frac{65794}{18}$ to clear of fractions, H′ = $\frac{6578\cdot4}{6579\cdot4}\times90\times\frac{65794}{18}$ or 328920 teeth, and E′ = $54\times\frac{65794}{18}\times197382$ teeth. The factors of these numbers are too large for compound gears; therefore, the numbers are reduced by the principle of continued fractions to other numbers from which suitable factors can be obtained.

Thus,

328920	197382	1
197382	131538	1
131538	65844	1
65844	65694	1
65694	150	437
65550	144	1
144	6	24
144		

									24 3
		1	1	1	1	437	1		5261
1	0	1	1	2	3	1313	1316		8767
0	1	1	2	3	5	2188	2193		5267
		1	1	1	1	437	1		8777
									24 5

Since $\frac{197382}{328920}$ = approximately $\frac{1316}{2193}$, E′ = 1316 teeth, and H′ = 2193 teeth. After factoring (1316 = 28×47; 2193 = 43×51) and placing in proper compound order, E = 28 teeth, F = 43 teeth, G = 47 teeth, and H = 51 teeth.

Check: When E = 28 teeth, F = 43 teeth, G = 47 teeth, and H = 51 teeth, the work will make $54\times\frac{43}{28}\times\frac{51}{47}\times\frac{1}{90}$ or $\frac{6579}{6580}$ of a revolution while the hob makes 54 revolutions. Therefore, the intersecting points of the sucessive turns of the feed spiral are out of line $1-\frac{6579}{6580}$ or $\frac{1}{6580}$ of a complete circle along any tooth spiral being considered. From this it can be seen that the tooth spiral crosses 6580 spaces between the turns of the feed spiral in one complete revolution, while a straight line will cross 6580 − 1 or 6579 spaces in the same distance of the tooth spiral.

If one space of the feed spiral measured in a straight line equals .022″, 6579 spaces must equal 6579×.022″ or 144.738″, which varies only .009″ from the true lead. The variation for a gear one inch wide would be $\frac{.009}{144}$ or .00006″.

SOLUTION OF THE FEED GEARS.

When the feed gears A, B, C, and D all have the same number of teeth, the head carrying the hob will feed .125″; therefore, the feed gear ratio for a .020″ feed is $\frac{.125}{.020}=\frac{125}{20}=\frac{5\times25}{2\times10}=\frac{60\times50}{24\times20}$.

Then, A = 60 teeth, B = 24 teeth, C = 50 teeth, and D = 20 teeth.

2. When the feed gears A, B, C, and D all have the same number of teeth, the head carrying the hob will feed .125″; therefore, when A = 72 teeth, B = 30 teeth, C = 50 teeth, and D = 20 teeth, it will feed $.125''\times\frac{20\times30}{72\times50}$ or $.02\frac{1}{12}''$ or .0208333″.

When E = 36 teeth, F = 74 teeth, G = 24 teeth, and H = 39 teeth, the work will make $27\times\frac{39}{24}\times\frac{74}{36}\times\frac{1}{90}$ or $\frac{481}{480}$ revolutions while the hob makes 27 revolutions.

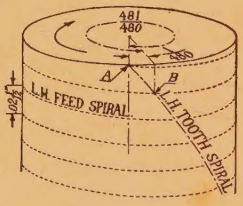


FIG. 487S

Since a left-hand hob is being used, the feed spiral must be left-handed. If a left-hand hob feeds down through the face of the work, the work must revolve as shown by the large arrow; and when traveling from A to B along the feed spiral, $\frac{481}{480}$ or $1\frac{1}{480}$ revolutions are required. Therefore, the intersecting points of the successive turns of the feed spiral are out of line $1\frac{1}{480}-1$ or $\frac{1}{480}$ of a complete circle along any tooth spiral being considered. From this it can be seen that the tooth spiral crosses 480 spaces between the turns of the feed spiral in one complete revolution; therefore, a straight line will cross 480+1 or 481 spaces in the same distance of the tooth spiral.

If one space between the feed spirals measures $.02\frac{1}{12}''$ or .0208333″, 481 spaces will measure $481\times.0208333''$ or 10.0208″; therefore, under conditions stated in the problem the lead will be 10.0208″.

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